CAL AB/BC

§3.1 Deriv of function p. 165 (1, 4, 7, 9, 10, 12-17, 20, 21, 22, 26, 28, 29 a,b only, 32, 36, 37, 42, 44)

Korpi
Period $\sqrt{-1}$

(1) $f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$

$f(x) = 1/x,\ a = 2$

$f'(2) = \lim_{h\to 0} \frac{f(2+h) - f(2)}{h}$

$= \lim_{h\to 0} \frac{\frac{1}{2+h} - \frac{1}{2}}{h}\left(\frac{(2+h)(2)}{(2+h)(2)}\right)$

$= \lim_{h\to 0} \frac{\cancel{2} - \cancel{2} - h}{2h(2+h)}$

$= \lim_{h\to 0} \frac{-\cancel{h}}{2\cancel{h}(2+h)}$

$= \boxed{\frac{-1}{4}}$

(4) $f(x) = x^3 + x,\ a = 0$

$f'(0) = \lim_{h\to 0} \frac{f(0+h) - f(0)}{h}$

$= \lim_{h\to 0} \frac{h^3 + h - 0}{h}$

$= \lim_{h\to 0} \frac{\cancel{h}(h^2+1)}{\cancel{h}}$

$= \boxed{1}$

(7) $f(x) = \sqrt{x+1},\ a = 3$

$f'(3) = \lim_{x\to 3} \frac{f(x) - f(3)}{x-3}$

$= \lim_{x\to 3} \frac{\sqrt{x+1} - 2}{x-3}\left(\frac{\sqrt{x+1}+2}{\sqrt{x+1}+2}\right)$

$= \lim_{x\to 3} \frac{(x+1-2)}{(x-3)(\sqrt{x+1}+2)}$

$= \boxed{\frac{1}{4}}$

(9) $f'(x)$ if $f(x) = 3x - 12$

$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h\to 0} \frac{[3(x+h) - 12] - [3x-12]}{h}$

$= \lim_{h\to 0} \frac{\cancel{3x} + 3h - \cancel{12} - \cancel{3x} + \cancel{12}}{h}$

$= \lim_{h\to 0} \frac{3\cancel{h}}{\cancel{h}}$

$= \boxed{3}$

(10) $\frac{dy}{dx}$ if $y = 7x$

$\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h\to 0} \frac{7(x+h) - 7x}{h}$

$= \lim_{h\to 0} \frac{\cancel{7x} + 7h - \cancel{7x}}{h}$

$= \lim_{h\to 0} \frac{7\cancel{h}}{\cancel{h}}$

$= \boxed{7}$

(12) $\frac{d}{dx} f(x)$ if $f(x) = 3x^2$

$\frac{d}{dx} f(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

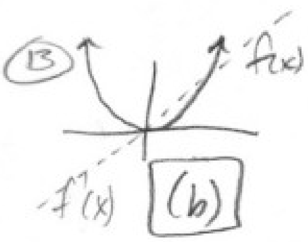
$\lim_{h\to 0} \frac{3(x+h)^2 - 3x^2}{h}$

$= \lim_{h\to 0} \frac{\cancel{3x^2} + 6xh + 3h^2 - \cancel{3x^2}}{h}$

$= \lim_{h\to 0} \frac{\cancel{h}(6x + 3h)}{\cancel{h}}$

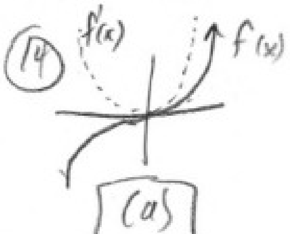
$= \boxed{6x}$

(13)



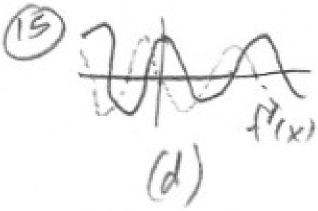
$\boxed{(b)}$

(14)



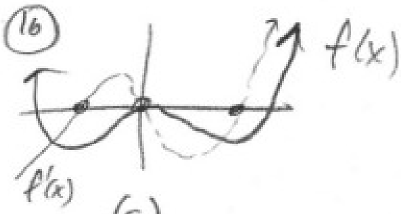
$\boxed{(a)}$

(15)



(d)

(16)



(c)

(17) $f(2) = 3,\ f'(2) = 5 = m$

a) tan line @ $x = 2$

$y - 3 = 5(x-2)$

$y = 5x - 10 + 3$

$\boxed{y = 5x - 7}$

b) Normal line, $m = -\frac{1}{5}$

$y - 3 = -\frac{1}{5}(x-2)$

$y = -\frac{1}{5}x + \frac{2}{5} + 3$

$\boxed{y = -\frac{1}{5}x + \frac{17}{5}}$

(20) Find tan (a), Normal (b) lines to $y = \sqrt{x}$ @ $x = 4$

$\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h\to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}\left(\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}\right)$

$= \lim_{h\to 0} \frac{x + h - \cancel{x}}{h(\sqrt{x+h}+\sqrt{x})}$

$= \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h}+\sqrt{x})}$

$= \frac{1}{2\sqrt{x}}$

a) At $x = 4$

$\left.\frac{dy}{dx}\right|_{x=4} = \frac{1}{2\sqrt{4}} = \frac{1}{4}$

tan line: $y - 2 = \frac{1}{4}(x-4)$

$\boxed{y = \frac{1}{4}x + 1}$

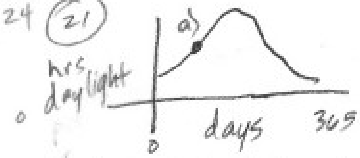
b) At $x = 4$, $m_\perp = -4$

Normal line: $y - 2 = -4(x-4)$

$\boxed{y = -4x + 18}$

Cal AB/BC
§3.1 cont

24 (21)



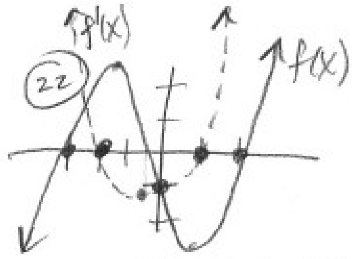
a) Inc at fastest rate at inflection pt a) $\approx$ April 1st

Slope $\approx \frac{3}{15} \approx \frac{1}{5}$ hr/day

b) Yes, Jan 1, July 1, this is where graph turns (Relative extrema)

c) Rate of change (slopes) are positive through July 1st. and negative after July 1st.

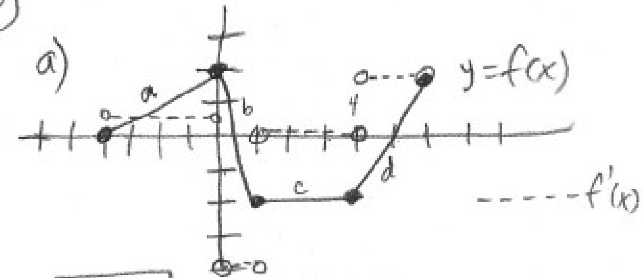
(22)



* the relative extrema (turning pts) of f(x) are the zeros of f'(x). The Inflection point of f(x) is the Relative min. of f'(x)

(26)

a)



Slopes

a: $\frac{2-0}{0--4} = \frac{2}{4} = \frac{1}{2}$

b: $\frac{2--2}{0-1} = \frac{4}{-1} = -4$

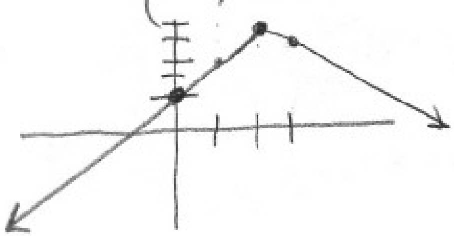
c: 0

d: $\frac{-2-2}{4-6} = \frac{-4}{-2} = 2$

b) f(x) is not differentiable when line segments change and endpts. @ $x = -4, 0, 1, 4, 6$

(28) Sketch continuous function f, Such that $f(0) = 1$

$f'(x) = \begin{cases} 2, & x < 2 \\ -1, & x > 2 \end{cases}$



(29) Skiing: $0 \le t \le 10$

a) dist (ft) / time (sec): the derivative gives the velocity of the skier in ft/sec at any given time, t.

b) feet/second (units of slope)

(32) $f(x) = \begin{cases} x^3, & x \le 1 \\ 3x, & x > 1 \end{cases}$

Left:

$\lim_{x \to 1^-} \frac{x^3 - (1)^3}{x-1}$

$= \lim_{x \to 1^-} \frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{(x-1)}}$

$= 3$

Right

$\lim_{x \to 1^+} \frac{3x-3}{x-1}$

$\lim_{x \to 1^+} \frac{3\cancel{(x-1)}}{\cancel{(x-1)}}$

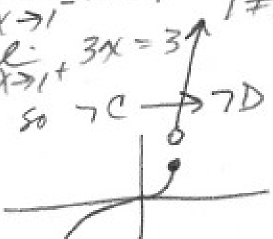
$= 3$

the slopes are equal, but since D $\to$ C, the limits must also equal:

$\lim_{x \to 1^-} x^3 = 1$ $\quad 1 \ne 3$

$\lim_{x \to 1^+} 3x = 3$

so $\neg C \to \neg D$



Note: we should use f(1) for our Right-sided limit:

$\lim_{x \to 1^+} \frac{f(x) - f(1)}{x-1}$

$\lim_{x \to 1^+} \frac{3x-1}{x-1}$

$= \frac{2}{0} \to$ VA

$\Rightarrow$ Right side deriv does not exist, so deriv does not exist

(36) T or F

if $f(x) = x^2 + x$, then $f'(x)$ exists $\forall x$.

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim \frac{(x+h)^2 + (x+h) - x^2 - x}{h}$

$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 + \cancel{x} + h - \cancel{x^2} - \cancel{x}}{h}$

$= \lim_{h \to 0} \frac{\cancel{h}(2x + h + 1)}{\cancel{h}}$

$= 2x + 1$ $\leftarrow$ continuous $\forall \mathbb{R}$

TRUE

(37) False. The left and right hand deriv not only have to exist, but they must be equal, AND the limits of the function values must also be the same (see prob 32)

(42) $f(x) = \begin{cases} x^2, & x \le 1 \\ 2x, & x > 1 \end{cases}$

a) $f'(x) = 2x$, $x < 1$

b) $f'(x) = 2$, $x > 1$

c) $\lim_{x \to 1^-} f'(x) = 2$

d) $\lim_{x \to 1^+} f'(x) = 2$

e) $\lim_{x \to 1} f'(x)$ exists $(= 2)$ since 2 one-sided limits were equal

f) $\lim_{x \to 1^-} \frac{x^2 - 1^2}{x-1}$

$\lim_{x \to 1^-} \frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}}$

$= 2$

g) $\lim_{x \to 1^+} \frac{f(x) - f(1)}{x-1}$ $\leftarrow$ use where it is defined

$\lim_{x \to 1^+} \frac{2x-1}{x-1} = \frac{1}{0} \to$ VA

= DNE

h) So f(x) is NOT diff'able at $x = 1$

(44) $f(x) = \begin{cases} x^3, & x \le 1 \\ 3x + k, & x > 1 \end{cases}$

find k to make diff'able at $x = 1$

continuous

i) $f(1) = 1$

ii) $\lim_{x \to 1^-} f(x) = 1$

$\lim_{x \to 1^+} f(x) = 3 + k$

$3 + k = 1$

$k = -2$

continuous

check diff'able

$f'(x) = \begin{cases} 3x^2, & x < 1 \\ 3, & x > 1 \end{cases}$

$\lim_{x \to 1^-} f'(x) = 3$

$\lim_{x \to 1^+} f'(x) = 3$

slopes are same AND continuous if $\boxed{k = -2}$

p. 2/2 – Koz