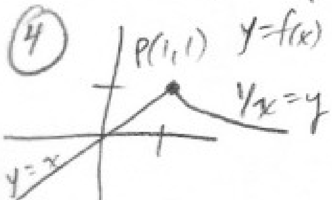
Cal AB/BC

§3.2 p.114 (4, 8, 10, 11-14, 20, 28, 30, 35, 39, 40, 41, 47, 48)

Korpi
Period $\sqrt{-1}$

④




Not diff'able at $x=1$ (Cusp)

$\lim_{x\to 1^-} f'(x) = 1$

$\lim_{x\to 1^+} f'(x) = -$

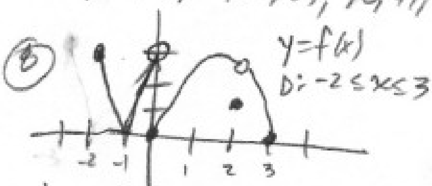
$\lim_{h\to 0} \frac{\frac{1}{1+h}-\frac{1}{1}}{h}\left(\frac{1+h}{1+h}\right)$

$=\lim_{h\to 0} \frac{1-1-h}{h(1+h)}$

$=\lim_{h\to 0} \frac{-1}{(1+h)} = -1$

$\lim_{x\to 1^-} f'(x) \neq \lim_{x\to 1^+} f'(x)$

⑧

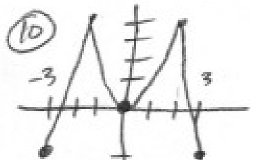



a) Diff'able at all pts in [-2,3] except $x=-2,-1,0,2,3$

b) Cont but Not diff'able at $x=-2,-1,3$

c) Neither cont nor diff. at $x=0, x=2$

⑩

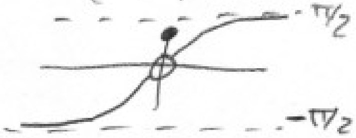



a) Diff'able at all pts in [-3,3] except $x=-3,-2,2,3$

b) Cont. but not Diff'able at $x=-3,-2,2,3$

c) Neither cont. nor diff. at No pts in [-3,3]

⑪ $y=\begin{cases}\tan^{-1}x, & x\neq 0\\ 1, & x=0\end{cases}$




Not diff'able at $x=0$ because of the discontinuity

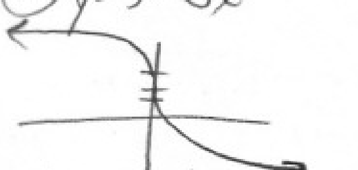
⑫ $y=x^{4/5}$

Not diff'able @ $x=0$ because of a cusp

⑬ $y=x+\sqrt{x^2}+2$
$=x+|x|+2$

Not diff'able at $x=0$ because of a sharp turn.

⑭ $y=3-\sqrt[3]{x}$



Not diff'able at $x=0$ because of a Vertical tangent line

⑳ $f(x)=x^3-4x$ @ $x=0$

Numerical deriv, $h=0.001$

$\frac{f(0+0.001)-f(0)}{0.001-0}$

$\frac{-0.0039999-0}{0.001}$

$=-3.9999$

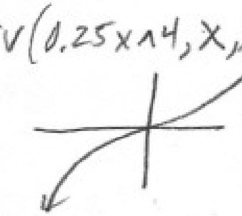
$\approx -4$

→ So $f(x)$ is diff'able @ $x=0$

㉘ $y=0.25x^4$

y1: nderiv(0.25x^4,X,X)

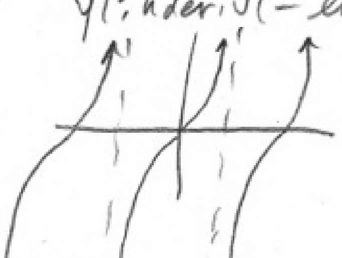
[Math] [#8]



resembles $y=x^3$

㉚ $y=-\ln|\cos x|$

y1: nderiv(-ln(cos(X)),X,X)



resembles $y=\tan x$

㉟ $g(x)=\begin{cases}(x+1)^2, & x\le 0\\ 2x+1, & 0<x<3\\ (4-x)^2, & x\ge 3\end{cases}$

✱ the 3 pieces themselves are polynomials and are diff'able over their defined intervals. We need to check the pts. of exchange: $x=0, x=3$

@ $x=0$

Continuous?

$\lim_{x\to 0^-} g(x)=1$ ✓

$\lim_{x\to 0^+} g(x)=1$ ✓

diff'able? $\lim_{x\to 0^-} g'(x)=2$ ✓

$\lim_{x\to 0^+} g'(x)=2$ ✓

@ $x=3$

Continuous

$\lim_{x\to 3^-} g(x)=7$

$\lim_{x\to 3^+} g(x)=-5$

Not continuous So Not diff'able @ $x=3$

㊴ $f(x)=\begin{cases}3-x, & x<1\\ ax^2+bx, & x\ge 1\end{cases}$

a) to be continuous @ $x=1$

i) $\lim_{x\to 1^-} f(x)=2$

$\lim_{x\to 1^+} f(x)=a+b$

So $\boxed{a+b=2}$

b) to be diff'able @ $x=1$

$\lim_{x\to 1^-} f'(x)=\lim_{x\to 1^+} f'(x)$

$\boxed{-1=2a+b}$

Solve system of equations

$a=2-b$ So $-1=2(2-b)+b$

$-1=4-2b+b$

$-5=-b$

$\boxed{b=5}$

$a=2-5$

$\boxed{a=-3}$

(pg. 1/2)

(Cal AB/BC)

§3.2 cont

(40) (T) or F

If f is diff'able at $x=a$, then it is continuous!

$(D \rightarrow C)$

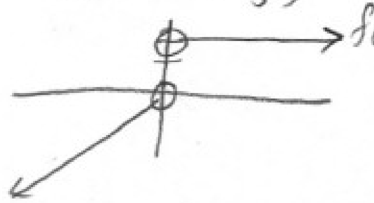
(47) $f(x)=\begin{cases} x^2, & x\le 0 \\ 2x, & x>0 \end{cases}$

a)

b) it only graphs the two equations in the specified intervals of $x$

c) Y2: nderiv(Y1, X, X)

(turn off $y_1$)




d) NDeriv(Y1, X, −0.1) = −0.2

NDeriv(Y1, X, 0) = 0.9995

NDeriv(Y1, X, 0.1) = 2

from ex 6) p.104

Nderiv(Y1, X, −0.1) ≈ 0

Nderiv(Y1, X, 0.1) ≈ 2

Nderiv(Y1, X, 0) = DNE

* we must know when to Distrust our calculator!!

(It tries to connect the two curves @ x=0)

(41) T or (F)

if f is cont @ x=a it is diff'able at x=a.

ex) $y=|x|$ @ $x=0$ sharp turn

$y=x^{1/3}$ @ $x=0$ cusp

$y=x^{2/3}$ @ $x=0$ vert tan

(48) $f(x)=\begin{cases} x\sin\frac{1}{x}, & x\neq 0 \\ 0, & x=0 \end{cases}$ oscillation

a) @ $x=0$

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = 0$ (by calculator)

$f(0)=0$ so $f(x)$ is continuous @ $x=0$

b) show $\frac{f(0+h)-f(0)}{h} = \sin\frac{1}{h}$

$= \frac{h\sin\frac{1}{h} - 0}{h} = \boxed{\sin\frac{1}{h}}$

c) $\lim_{h\to 0} \frac{f(0+h)-f(0)}{h} = DNE$

since $\lim_{h\to 0} \sin\frac{1}{h} = \sin\left(\frac{1}{0}\right)$

$=\sin(\infty)$ which oscillates between −1 and 1.

d) No, the derivative does not exist from either side of $x=0$ since the limit did not exist in part c)

e) $g(x)=\begin{cases} x^2\sin\frac{1}{x}, & x\neq 0 \\ 0, & x=0 \end{cases}$, show $g'(0)=0$

$g'(0)=\lim_{h\to 0} \frac{(0+h)^2\sin\frac{1}{0+h} - 0^2\sin\left(\frac{1}{0}\right)}{h}$

$=\lim_{h\to 0} \frac{h^2\cdot\sin\left(\frac{1}{h}\right)}{h} = \lim_{h\to 0} h\cdot\sin\left(\frac{1}{h}\right)$

$=0\cdot\sin\left(\frac{1}{0}\right) = \boxed{0}$

this oscillates between y=−1, y=1, so Zero times these y-values always is zero!

 — Koji