ABC Cal | §3.3 | p. 124 (6, 10, 14, 19, 24, 26, 28, 31, 32, 38, 39, 46, 47, 53, 54, AP 1–4

Korpi
period $\sqrt{-1}$

(6) $y = 1 - x + x^2 - x^3$

$\frac{dy}{dx} = y' = \boxed{-1 + 2x - 3x^2}$

(10) $y = 4x^3 - 6x^2 - 1$

Horz tan when $y' = \frac{dy}{dx} = 0$

$\frac{dy}{dx} = 12x^2 - 12x = 0$

$12x(x-1) = 0$

$x = 0, x = 1$

So Horz ~~tangents~~ at

$\boxed{(0, y(0)) = (0, -1) \quad (1, y(1)) = (1, -3)}$

(14) $y = \frac{(x^2+3)}{x}$

a) $\frac{dy}{dx} = y' = \frac{x(2x) - (x^2+3)(1)}{x^2} = \boxed{\frac{x^2-3}{x^2}}$

b) $y = \frac{x^2}{x} + \frac{3}{x} = x + 3x^{-1}$

$\frac{dy}{dx} = y' = 1 - 3x^{-2} = 1 - \frac{3}{x^2}$

$= \boxed{\frac{x^2-3}{x^2}}$

(19) $y = \frac{(x-1)(x^2+x+1)}{x^3}$

simplify 1st!!

$y = (x^3 + x^2 + x - x^2 - x - 1)(x^{-3})$

$y = (x^3 - 1)(x^{-3})$

$y = 1 - x^{-3}$

$\frac{dy}{dx} = y' = 3x^{-4} = \boxed{\frac{3}{x^4}}$

(24) $u(2) = 3, u'(2) = -4$

$v(2) = 1, v'(2) = 2$

@ $x = 2$

a) $\frac{d}{dx}(uv) = u'v + uv'$

$\frac{d}{dx}(uv)\Big|_{x=2} = (-4)(2) + (3)(2)$

$= -8 + 6$

$= \boxed{-2}$

(24b) $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \cdot u' - u \cdot v'}{v^2}$

$\frac{d}{dx}\left(\frac{u}{v}\right)\Big|_{x=2} = \frac{1(-4) - 3(2)}{1^2}$

$= -4 - 6$

$= \boxed{-10}$

(24c) $\frac{d}{dx}\left(\frac{v}{u}\right) = \frac{u \cdot v' - v \cdot u'}{u^2}$

$\frac{d}{dx}\left(\frac{v}{u}\right)\Big|_{x=2} = \frac{3(2) - (1)(-4)}{3^2}$

$= \frac{6+4}{9} = \boxed{\frac{10}{9}}$

(24d) $\frac{d}{dx}(3u - 2v + 2uv)$

$= 3u' - 2v' + 2uv' + 2u'v$

$\frac{d}{dx}\Big|_{x=2} = 3(-4) - 2(2) + 2(3)(2) + 2(-4)(1)$

$= -12 - 4 + 12 - 8$

$= \boxed{-12}$

(26) slope of

$3x - 2y + 12 = 0$

$-2y = -3x - 12$

$y = \frac{3}{2}x + 6$

$\boxed{m = \frac{3}{2}}$ iii

(28) $y = \frac{x^4 + 2}{x^2}$ @ $x = -1$

$y = x^2 + 2x^{-2}$

$y' = \frac{dy}{dx} = 2x - 4x^{-3} = 2x - \frac{4}{x^3}$

$y'(-1) = 2(-1) - \frac{4}{(-1)^3} = -2 + 4 = \boxed{2 = m}$

$(-1, y(-1)) = (-1, 3)$

eq. of tan line:

$y - 3 = 2(x+1)$

$\boxed{y = 2x + 5}$

(31) $y = \frac{\sqrt{x} - 1}{\sqrt{x} + 1}$

$y' = \frac{dy}{dx} = \frac{(\sqrt{x}+1)(\frac{1}{2}x^{-1/2}) - (\sqrt{x}-1)(\frac{1}{2}x^{-1/2})}{(\sqrt{x}+1)^2}$

$= \frac{\frac{\sqrt{x}+1}{2\sqrt{x}} - \frac{\sqrt{x}-1}{2\sqrt{x}}}{(\sqrt{x}+1)^2}$

$\boxed{y' = \frac{x-1}{2\sqrt{x}(\sqrt{x}+1)^2}}$

(32) $y = 2\sqrt{x} - \frac{1}{\sqrt{x}}$

$y = 2x^{1/2} - x^{-1/2}$

$y' = \frac{dy}{dx} = x^{-1/2} + \frac{1}{2}x^{-3/2}$

$\boxed{y' = \frac{1}{\sqrt{x}} + \frac{1}{2\sqrt{x^3}}}$

(38) $y = x^3 + x$, where is slope 4.

$y' = 3x^2 + 1 = 4$

$3x^2 = 3$

$\boxed{x = \pm 1}$

at $x = 1$: $y - 2 = 4(x-1)$

$\boxed{y = 4x - 2}$

at $x = -1$: $y + 2 = 4(x+1)$

$\boxed{y = 4x + 2}$

slope is smallest when $y' = 3x^2 + 1$ is minimized which is at $x = 0$, $y'(0) = 1$

ABC Cal §3.3 cont

39 $y = 2x^3 - 3x^2 - 12x + 20$

Find where parallel to x-axis

$y' = \frac{dy}{dx} = 6x^2 - 6x - 12$

slope of x-axis $= 0$, so $y' = 0$

$6x^2 - 6x - 12 = 0$

$6(x^2 - x - 2) = 0$

$6(x-2)(x+1) = 0$

$x = 2, -1$

$\boxed{\text{points: } (-1, y(-1)) = (-1, 27) \quad (2, y(2)) = (2, 0)}$

47 $s = 4.9t^2$ m, sec

$\frac{ds}{dt} = v(t) = 9.8t$ m/sec

$\frac{d^2s}{dt^2} = v'(t) = a(t) = 9.8$ m/sec^2

46 $P = \frac{nRT}{V - nb} - \frac{an^2}{V^2} = \frac{nRT}{V-nb} - (an^2)V^{-2}$

a, b, n, R are constants!!

$\frac{dP}{dV} = ?$

$\frac{dP}{dV} = \frac{(V-nb)(0) - (nRT)(1)}{(V-nb)^2} + 2(an^2)V^{-3}$

$$\boxed{\frac{dP}{dV} = \frac{-nRT}{(V-nb)^2} + \frac{2an^2}{V^3}}$$

53 T or F: $\frac{d}{dx}(\pi^3) = 3\pi^2 \rightarrow$ FALSE

π^3 is a constant, its rate of change is ZERO!!

54 T or F: $f(x) = \frac{1}{x}$ has No Horz. tans.

$f'(x) = -\frac{1}{x^2} \neq 0$, so TRUE

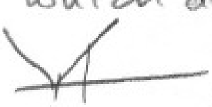
AP1 M.C. $f(x) = |x+1|$

I. f cont @ $x = -1$

II. f diff'able @ $x = -1$

III. f has corner @ $x = -1$

which are true



, I and III , $\boxed{D}$

AP2 M.C. Normal line passes through $(1,2)$ and $(-1,1)$

so $m_\perp = \frac{1-2}{-1-1} = \frac{-1}{-2} = \frac{1}{2}$

So $m_{tan} = f'(1) = -2$ (opp recip)

$\boxed{A}$

AP3 $y = \frac{4x-3}{2x+1}$

$\frac{dy}{dx} = \frac{(2x+1)(4) - (4x-3)(2)}{(2x+1)^2}$

$= \frac{8x+4-8x+6}{(2x+1)^2}$

$= \frac{10}{(2x+1)^2}$ $\boxed{C}$

AP4 $f(x) = x^4 - 4x^2$

a) Horz tans when $f'(x) = 0$

$f'(x) = 4x^3 - 8x = 0$

$4x(x^2 - 2) = 0$

$\boxed{x = 0, \pm\sqrt{2}}$

b) tan line @ $x = 1$

$m = f'(1) = -4$

$(1, f(1)) = (1, -3)$

$y + 3 = (-4)(x-1)$

$\boxed{y = -4x + 1}$

c) Normal line @ $x = 1$

$m = \frac{1}{4}, (1, -3)$

$y + 3 = \frac{1}{4}(x-1)$

$\boxed{y = \frac{1}{4}x - \frac{13}{4}}$

-Koji