Cal ABC   §3.4 p.135(3,8,9,10,11,13,20,24,26,29,32,36,40,46,47)

-Korpi
Period $\sqrt{-1}$

③ a) $A(s) = \frac{\sqrt{3}}{4}s^2 \rightarrow$ Area of Eq. △

b) $A'(s) = \frac{dA}{ds} = \frac{\sqrt{3}}{2}s$

c) $A'(2) = \sqrt{3}$

$A'(10) = 5\sqrt{3}$

d) $\frac{dA}{ds} = in^2/in$

So $A'(2)$ means that when the side length is 2 inches, the Area is increasing at a rate of $\sqrt{3}$ square inches per inch of side length increase.

⑧ $Q(t) = 200(30-t)^2$

$Q(t)$ = gal. in tank.

t = time in min

How fast is water running out at 10 min?

$Q(t) = 200(900 - 60t + t^2)$

$Q'(t) = 200(-60 + 2t)$

$Q'(10) = 200(-60 + 20)$

neg means "decreasing" $= -8000$ gal/min

$\boxed{\text{So At 10 min, water is running out at 8000 gallons per min}}$

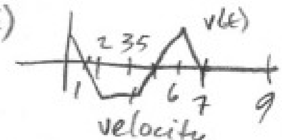
Avg rate of change $t: [0,10]$

$= \frac{Q(10) - Q(0)}{10 - 0} = \frac{80000 - 180000}{10}$

$= \frac{-100000}{10} = -10{,}000$ gal/min

$\boxed{\text{So the Average rate at which tank drained in the first 10 min. was 10,000 gallons per minute}}$

⑨ $v(t) = f(t)$



a) Particle moves
- forward when $v(t) > 0$ on $(0,1) \cup (5,7)$
- backward when $v(t) < 0$ $(1,5)$

Slows down when $v(t) \rightarrow 0$ $(3,5) \cup (6,7)$

b) accel > 0 → Slopes of graph of $v(t)$ are pos: $(3,6)$

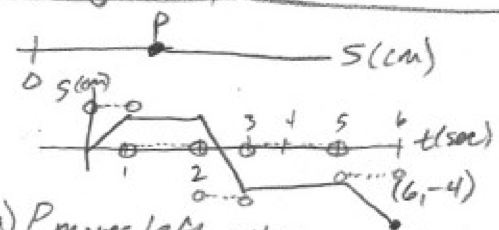
accel < 0 → neg slope of $v(t)$ $(0,2) \cup (6,7)$

zero accel when zero slope $(2,5) \cup (7,9)$

c) greatest speed when furthest from x-axis → $(2,3)$

d) Particle is still when $v(t) = 0$ $(7,9)$

⑩



a) P moves left when graph of s moves down (neg slope → neg vel) $(2,3) \cup (5,6)$

moves right $(0,1)$

stands still $(1,2) \cup (3,5)$

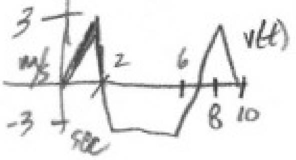
b) Velocity: dotted line above

c) Speed = |velocity| so just reflect all neg horz line segments above x-axis.

*Note: since y-axis is not scaled, we don't know the true values of velocity (slopes of s) only that they are pos/neg/zero.

Cal ABC §3.4 cont

(11) velocity $= \frac{ds}{dt}$

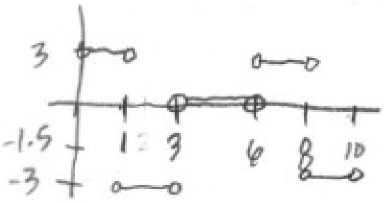


a) changes direction at a sign change $\boxed{@ t = 2, 7}$

b) moves at a constant rate at x (3,6)

c) Graph of speed $= |v(t)|$

d) accel is graph of slopes of $v(t)$



(13) Moon: $V_0 = 24$ m/sec

Height $s = 24t - 0.8t^2$

a) $v(t) = s'(t) = 24 - 1.6t$ m/sec

$a(t) = v'(t) = s''(t) = -1.6$ m/sec^2

b) Rock reaches high pt when $v(t) = 0$

$24 = 1.6t$

$\boxed{t = 15 \text{ seconds}}$

c) Rock hit a high of

$S(15) = 24(15) - 0.8(15^2)$

$= \boxed{180 \text{ meters}}$

d) Rock reached half its max height when $s(t) = 90$

$\boxed{t \approx 4.393 \text{ sec}}$

e) How long was Rock in "air?"

when $S(t) = 0$

$\boxed{t = 30 \text{ sec}}$

(20) Particle along a line, $t \geq 0$

$S(t) = -t^3 + 7t^2 - 14t + 8$

a) $\boxed{S'(t) = v(t) = -3t^2 + 14t - 14}$

b) $\boxed{v'(t) = a(t) = -6t + 14}$

c) Particle is at rest when $v(t) = 0$

$-3t^2 + 14t - 14 = 0$

calculator: $\boxed{t \approx 1.451, 3.215}$

d) Particle starts at position 8 and moves with neg velocity until $t = 1.451$ sec where it changes directions, then it changes directions again at $t = 3.215$ sec (where $v(t) = 0$ and changes sign).

(24) $v = 2t^3 - 9t^2 + 12t - 5$ m/sec

speed $= |v(t)|$

$a(t) = v'(t) = 6t^2 - 18t + 12 = 0$

$6(t^2 - 3t + 2) = 0$

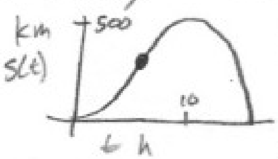
$6(t-2)(t-1) = 0$

accel $= 0$ @ $t = 2, 1$

Speed(1) $= |v(1)| = 0$ m/sec

speed(2) $= |v(2)| = 1$ m/sec

(26) truck, $0 \leq t \leq 15$



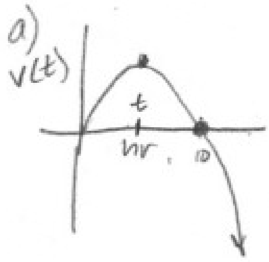
b) $S = 15t^2 - t^3$

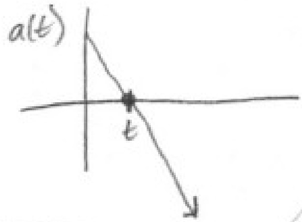
$S' = V = 30t - 3t^2$

$S'' = V' = a = 30 - 6t$

graphs are similar

a)



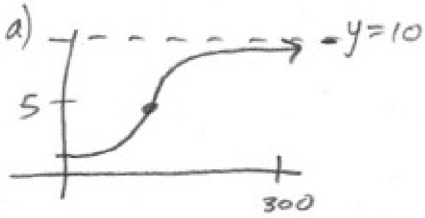


(29) $P(x) = \frac{10}{1 + 50 \cdot 2^{5 - 0.1x}}$

P → \$ thousands

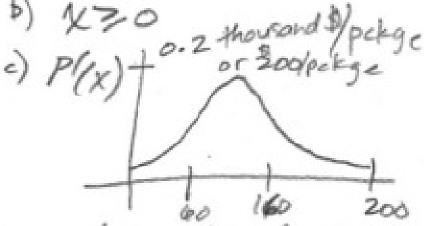
x = # packages sold

a)



b) $x \geq 0$

c)



P is most sensitive to change when $|P'(x)|$ is largest, x $60 < x < 160$

d) Marginal Profit $= P'(x)$ is greatest at $x = 106.44$ ($P''(x) = 0$) but since $x \in \mathbb{Z}$

$P(106) \approx 4.924$ thousand \$

or $\approx \boxed{\$4924}$

e) $P'(50) \approx 0.013$ or \$13/package sold

$P'(100) \approx 0.165$ or \$165/pckg

$P'(125) \approx 0.118$ or \$118/pckg

$P'(150) \approx 0.031$ or \$31/pcks

$P'(175) \approx 0.006$ or \$6/pckg

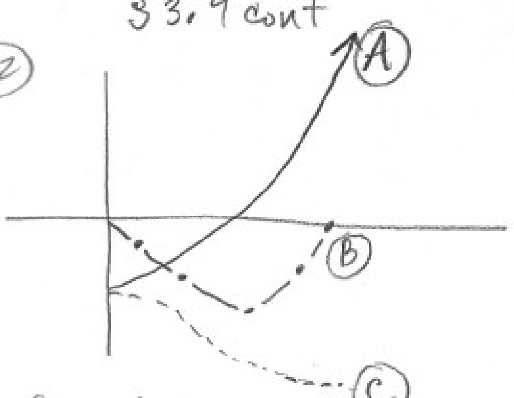
$P'(300) \approx 10^{-6}$ or \$0.001 per pckge

f) $\lim_{x \to \infty} P(x) = 10$ thousand \$, The max profit is \$10,000 per month

g) Yes, the company can sell as many as they can, but unless the lower/raise price, they won't increase or decrease their Profits.

CAL ABC
§3.4 cont

(32)



C → Position
B → Velocity
A → Acceleration

- the relative max/mins of f are the zeros of f'
- the inflection points of f are the relative max/mins of f' and the zeros of f''.

(36) From graph of velocity, to estimate the acceleration at a given point, just estimate the <u>SLOPE</u> of the velocity graph at that point.

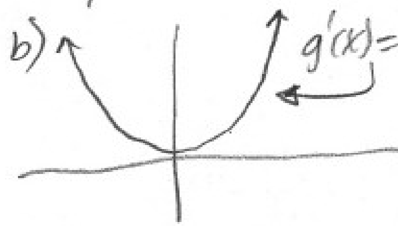
(40) T or F:
Speed at $t=a$ = Velocity @ $t=a$!
False, only if $v(t) \geq 0$.
In general speed $= |v(t)|$

(41) T or F:
$a(t) = s''(t)$
<u>True</u> $(= v'(t))$

(47) Finding f from f':
Let $f'(x) = 3x^2$

a) $g(x) = x^3$, $h(x) = x^3 - 2$, $t(x) = x^3 + 3$
$g'(x) = 3x^2$ $h'(x) = 3x^2$ $t'(x) = 3x^2$

b)



c) $f'(x) = 3x^2 \longrightarrow$ $\boxed{f(x) = x^3 + C \text{ where } C = \text{any constant}}$

d) $f'(x) = 3x^2$, $f(0) = 0$
$f(x) = x^3 + C$
$f(0) = 0^3 + C = 0$
$C = 0$
$\rightarrow \boxed{f(x) = x^3}$

e) $f'(x) = 3x^2$, $f(0) = 3$
$f(x) = x^3 + C$
$f(0) = 0^3 + C = 3$
$C = 3$
$\rightarrow \boxed{f(x) = x^3 + 3}$

– Korpi