Cal ABC §3.5 p. 146 (6, 16, 20, 23, 24, 30, 32, 36, 37, 38, 40, 41, 42, 44, 45, 46, 52) Korpi per. $\sqrt{-1}$

(6) $y = 3x + x\tan x$

$y' = \frac{dy}{dx} = 3 + \tan x + x\sec^2 x$

(16) $s = \cos t - 3\sin t$

a) $v(t) = s'(t) = -\sin t - 3\cos t$

speed $= |v(t)| = |-\sin t - 3\cos t|$

$a(t) = v'(t) = -\cos t + 3\sin t$

b) $v(\frac{\pi}{4}) = -\sin\frac{\pi}{4} - 3\cos\frac{\pi}{4}$

$= -\frac{\sqrt2}{2} - \frac{3\sqrt2}{2} = \frac{-4\sqrt2}{2} = \boxed{-2\sqrt2 \text{ m/sec}}$

speed$(\frac{\pi}{4}) = |-2\sqrt2| = \boxed{2\sqrt2 \text{ m/sec}}$

$a(\frac{\pi}{4}) = -\cos\frac{\pi}{4} + 3\sin\frac{\pi}{4}$

$= -\frac{\sqrt2}{2} + \frac{3\sqrt2}{2} = \frac{2\sqrt2}{2} = \boxed{\sqrt2 \text{ m/sec}^2}$

c) At $t = \frac{\pi}{4}$ the particle is moving in negative direction (neg velocity) and slowing down (v and a are opp signs at $t = \frac{\pi}{4}$)

(20) $s(t) = 2 + 2\sin t$

$v(t) = s'(t) = 2\cos t$

$a(t) = v'(t) = -2\sin t$

$j(t) = a'(t) = \boxed{-2\cos t}$

(23) $y = x^2 \sin x$ @ $x = 3$

$y' = \frac{dy}{dx} = 2x\sin x + x^2\cos x$

$y'(3) = \left.\frac{dy}{dx}\right|_{x=3} = 6\sin 3 + 9\cos 3 \approx -8.063$

$y(3) = 9\sin 3 \approx 1.270$

tan line

$y - 9\sin 3 = (6\sin 3 + 9\cos 3)(x-3)$

or $y = -8.063x + 25.460$

Normal line: $m_\perp = \frac{-1}{6\sin 3 + 9\cos 3}$

$y - 9\sin 3 = \left(\frac{-1}{6\sin 3 + 9\cos 3}\right)(x-3)$

or $y = 0.124x + 0.898$

(24) $\frac{d}{dx}[\cos x] = \lim_{h\to0} \frac{\cos(x+h) - \cos x}{h} = \lim_{h\to0} \frac{\cos x\cosh - \sin x\sinh - \cos x}{h}$

$= \lim_{h\to0} \frac{\cos x(\cos h - 1)}{h} - \frac{\sin x \cdot \sin h}{h}$

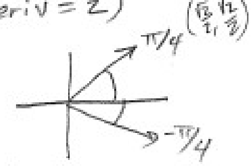
$= -\sin x$

(30) $y = \tan x \quad -\frac{\pi}{2} < x < \frac{\pi}{2}$ Find points where tangent line is parallel to $y = 2x$ (where slope/deriv = 2)

$y' = \frac{dy}{dx} = \sec^2 x = 2$

$\sec x = \sqrt2$ or $\sec x = -\sqrt2$

$\cos x = \frac{1}{\sqrt2} = \frac{\sqrt2}{2}$ or $\cos x = \frac{-1}{\sqrt2} = -\frac{\sqrt2}{2}$

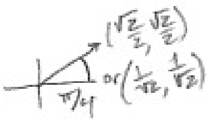


$y(\pi/4) = \tan\pi/4 = 1$

$y(-\pi/4) = \tan(-\pi/4) = -1$

The two points are $(\frac{\pi}{4}, 1)$ and $(-\frac{\pi}{4}, -1)$

(32) $y = 1 + \sqrt2\csc x + \cot x$



$y' = \frac{dy}{dx} = -\sqrt2\csc x\cot x - \csc^2 x$

a) $y'(\frac{\pi}{4}) = -\sqrt2\csc\frac{\pi}{4}\cot\frac{\pi}{4} - (\csc\frac{\pi}{4})^2$

$= -\sqrt2(\sqrt2)(1) - (\sqrt2)^2$

$= -2 - 2$

$= -4$

tan line $P(\frac{\pi}{4}, 4)$, $m = -4$

$y - 4 = -4(x - \frac{\pi}{4})$

or $y = -4x + \pi + 4$

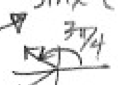
b) Horz tan @ Q, $m = y' = 0$

$-\sqrt2\csc x\cot x - \csc^2 x = 0$

$\csc x\left(-\sqrt2\left(\frac{\cos x}{\sin x}\right) - \frac{1}{\sin x}\right) = 0$

$\csc x = 0$ No soln or $\frac{1}{\sin x}(-\sqrt2\cos x - 1) = 0$

$-\sqrt2\cos x - 1 = 0$



$\cos x = -\frac{1}{\sqrt2} = -\frac{\sqrt2}{2}$

$x = \frac{3\pi}{4}$, $y = 1 + \sqrt2\csc\frac{3\pi}{4} + \cot\frac{3\pi}{4} = 2$

So Horz tan line is $\boxed{y = 2}$

(36) $y''(\theta) = \frac{d^2y}{d\theta^2}$

$y = \theta\tan\theta$

$y' = \frac{dy}{d\theta} = (1)\tan\theta + \theta\cdot(\sec^2\theta)$

$= \tan\theta + \theta\cdot\sec\theta\cdot\sec\theta$ triple product rule

$y'' = \frac{d^2y}{d\theta^2} = \sec^2\theta + (1)\sec^2\theta + \theta(\sec\theta\tan\theta)\sec\theta + \theta\cdot\sec\theta(\sec\theta\tan\theta)$

$= \boxed{2\sec^2\theta + 2\theta\sec^2\theta\tan\theta}$

Cal ABC §3.5

(37) $g(x) = \begin{cases} x+b, & x<0 \\ \cos x, & x \geq 0 \end{cases}$

Cont @ $x=0$:

i) $g(0) = \cos 0 = 1$ ✓

ii) $\lim_{x\to 0^-} g(x) = 0 + b = b$

$\lim_{x\to 0^+} g(x) = \cos 0 = 1$

So $\boxed{b=1}$

diff'able @ $x=0$

$g'(x) = \begin{cases} 1, & x<0 \\ -\sin x, & x>0 \end{cases}$

$\lim_{x\to 0^-} g'(x) = 1$

$\lim_{x\to 0^+} g'(x) = -\sin 0 = 0$

$1 \neq 0$

So NOT diff'able @ $x=0$

If $b=1$, $g(x)$ is continuous at $x=0$, but it can NOT be made differentiable there.

(38) $\frac{d^{999}}{dx^{999}}[\cos x]$

$y = \cos x$
$y' = -\sin x$
$y'' = -\cos x$
$y''' = \sin x$
$y^{(4)} = \cos x$

Repeats every 4th derivative

$999 \div 4 = 249$ R3

$$\begin{array}{r} 249 \\ 4\overline{)999} \\ \underline{8} \\ 19 \\ \underline{16} \\ 39 \\ \underline{36} \\ R3 \end{array}$$

So $\frac{d^{999}}{dx^{999}}[\cos x] = \frac{d^3}{dx^3}[\cos x]$

$= \boxed{\sin x}$

(40) $y = \sin x$

Near $x=0$, the graph of $\sin x$ resembles its tangent line (Locally Linear)

tan line:

$y' = \cos x$, $y'(0) = \cos 0 = 1 = m$

$y(0) = \sin(0) = 0$

$P(0,0)$, $m=1$

$y - 0 = 1(x-0)$

$\boxed{y = x}$

(41) a) $\sin(0.12) \approx y(0.12) = 0.12$ for $\boxed{y=x}$

b) $\sin(0.12) \approx 0.119712 \approx 0.12$

the approximation using the tangent line is within

$|0.119712 - 0.12| = 0.000288$

of actual answer

(42) $y = \sin 2x = 2\sin x\cos x$

$y' = (2\cos x)\cos x + 2\sin x(-\sin x)$

$= \boxed{2\cos^2 x - 2\sin^2 x}$

$= 2(\cos^2 x - \sin^2 x)$

$= 2\cos 2x$

So $\frac{d}{dx}[\sin(2x)] = 2\cdot\cos(2x)$

we'll do this more directly when we learn the Chain Rule.

(44) T or F:

$S(t) = -3\sin t$

$s'(t) = -3\cos t$

$s'\left(\frac{3\pi}{4}\right) = -3\cos\left(\frac{3\pi}{4}\right)$

$= -3\left(-\frac{\sqrt{2}}{2}\right) = +\frac{3\sqrt{2}}{2}$

So spring is moving in Pos direction.

True, spring is traveling upward

(45) $v\left(\frac{\pi}{4}\right) = s'\left(\frac{\pi}{4}\right) = -3\cos\frac{\pi}{4} = -\frac{3\sqrt{2}}{2}$

Neg Velocity

speed $= \left|v\left(\frac{\pi}{4}\right)\right| = +\frac{3\sqrt{2}}{2}$

pos speed (ALWAYS)

False, not the same

(46) $y = \sin x + \cos x$ @ $x = \pi$, tan line is...

$y' = \cos x - \sin x$

$y'(\pi) = \cos\pi - \sin\pi = -1$

$y(\pi) = \sin\pi + \cos\pi = -1$

$m = -1$, $P(\pi, -1)$

tan line

$y + 1 = -1(x - \pi)$

$\boxed{y = -x + \pi - 1}$

$\boxed{A}$

(52) $y = A\sin x + B\cos x$

$y' = A\cos x - B\sin x$

$y'' = -A\sin x - B\cos x$

$y'' - y = \sin x$ (plug in)

$-A\sin x - B\cos x - A\sin x - B\cos x = \sin x$

$-2A\sin x - 2B\cos x = \sin x$

(1 eq, 2 unknowns, No unique soln)

Find 1:

Let $\boxed{B=0}$ (to get rid of cosines)

$-2A\sin x = \sin x$

So $-2A = 1$

$\boxed{A = -\frac{1}{2},\ B = 0}$

- Koopi