Cal ABC §3.7 p.162 (1, 4, 7, 11, 16, 18, 24, 30–42 even, 43, 50, 57)

Karpi
period $\sqrt{-1}$

① $x^2y + xy^2 = 6$

$\frac{d}{dx}\left[x^2y + xy^2 = 6\right]$

$2xy + x^2\frac{dy}{dx} + y^2 + 2xy\frac{dy}{dx} = 0$

$\frac{dy}{dx}\left(x^2 + 2xy\right) = -2xy - y^2$

$\boxed{\frac{dy}{dx} = \frac{-2xy - y^2}{x^2 + 2xy} = -\frac{2xy + y^2}{2xy + x^2}}$

④ $x^2 = \frac{x-y}{x+y}$

$\frac{d}{dx}\left[x^2 = \frac{x-y}{x+y}\right]$

$2x = \frac{(x+y)(1-y') - (x-y)(1+y')}{(x+y)^2}$

$2x(x+y)^2 = x + y - y'(x+y) - x + y - y'(x-y)$

$y'(x+y+x-y) = x + y - x + y - 2x(x+y)^2$

$y' = \frac{2y - 2x(x+y)^2}{2x} = \boxed{\frac{y - x(x+y)^2}{x} = y'}$

forms may vary

⑦ $x + \tan(xy) = 0$

$\frac{d}{dx}\left[x + \tan(xy) = 0\right]$

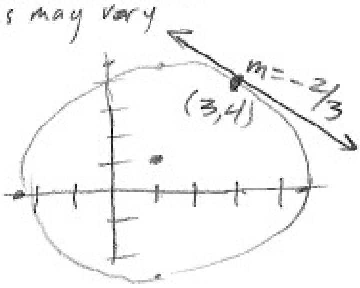
$1 + \sec^2(xy)\cdot(y + xy') = 0$

$1 + y\sec^2(xy) + xy'\sec^2(xy) = 0$

$y'(x\sec^2(xy)) = -1 - y\sec^2(xy)$

$\boxed{y' = -\frac{1 + y\sec^2(xy)}{x\sec^2(xy)}}$

⑪ $(x-1)^2 + (y-1)^2 = 13$, m @ $(3,4)$



$\frac{d}{dx}$: $2(x-1) + 2(y-1)\cdot y' = 0$

$y'(2y-2) = -2x + 2$

$y' = \frac{2-2x}{2y-2} = \boxed{\frac{1-x}{y-1} = y'}$

$y'(3,4) = \frac{1-3}{4-1} = \boxed{\frac{-2}{3} = m}$

⑯ $x^2 + 4xy + 4y^2 - 3x = 6$

$\frac{d}{dx}$: $2x + 4y + 4xy' + 8yy' - 3 = 0$

$y'(4x + 8y) = 3 - 2x - 4y$

$y' = \frac{3 - 2x - 4y}{4x + 8y}$

defined when $4x + 8y \neq 0$

$4x = -8y$

$\boxed{x = -2y}$

plug into eq:

$(-2y)^2 + 4(-2y)y + 4y^2 - 3(-2y) = 6$

$4y^2 - 8y^2 + 4y^2 + 6y = 6$

$y = 1,\ x = -2(1) = -2$

$\boxed{\frac{dy}{dx} \text{ is defined everywhere except @ } (-2, 1)}$

⑱ $x^2 + y^2 = 25$

$\frac{d}{dx}$: $2x + 2yy' = 0$

$y' = \frac{-2x}{2y}$

$\boxed{y' = -\frac{x}{y}}$

@ $(3,-4)$

$m = y'(3,-4) = \frac{3}{4}$

tan line

$y + 4 = \frac{3}{4}(x-3)$

or $y = \frac{3}{4}x - \frac{25}{4}$

$m_\perp = \frac{-4}{3}$

Normal line

$y + 4 = -\frac{4}{3}(x-3)$

or $y = -\frac{4}{3}x$

㉔ $x\sin(2y) = y\cos(2x)$

$\frac{d}{dx}$: $\sin(2y) + 2xy'\cos(2y) = y'\cos(2x) - 2y\sin(2x)$

$y'(2x\cos(2y) - \cos(2x)) = -2y\sin(2x) - \sin(2y)$

$y' = -\frac{2y\sin 2x + \sin 2y}{2x\cos 2y - \cos 2x}$ @ $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

$m = y'\left(\frac{\pi}{4}, \frac{\pi}{2}\right) = -\frac{2\left(\frac{\pi}{2}\right)\sin\frac{\pi}{2} + \sin\pi}{2\left(\frac{\pi}{4}\right)\cos\pi - \cos\frac{\pi}{2}} = \frac{-\pi}{\frac{\pi}{2}(-1)} = \boxed{2 = m}$

tan line

$y - \frac{\pi}{2} = 2\left(x - \frac{\pi}{4}\right)$

or $\boxed{y = 2x}$

$m_\perp = -\frac{1}{2}$

Normal line

$y - \frac{\pi}{2} = -\frac{1}{2}\left(x - \frac{\pi}{4}\right)$

or $\boxed{y = -\frac{1}{2}x + \frac{5\pi}{8}}$

(30) $y^2+2y=2x+1$

$\frac{d}{dx}: 2yy'+2y'=2$

$y'(2y+2)=2$

$\boxed{y'=\frac{dy}{dx}=\frac{1}{y+1}}$

$\frac{d^2}{dx^2}: \quad y''=\frac{(y+1)(0)-1(y')}{(y+1)^2}$

$y''=\frac{-y'}{(y+1)^2}=\frac{-\left(\frac{1}{y+1}\right)}{(y+1)^2}$

$\boxed{y''=\frac{d^2y}{dx^2}=\frac{-1}{(y+1)^3}}$

(32) $y=x^{-3/5}$

$\frac{dy}{dx}=-\frac{3}{5}x^{-8/5}$

$\boxed{\frac{dy}{dx}=\frac{-3}{5x^{8/5}}}$

(34) $y=\sqrt[4]{x}=x^{1/4}$

$y'=\frac{dy}{dx}=\frac{1}{4}x^{-3/4}$

$\boxed{\frac{dy}{dx}=\frac{1}{4x^{3/4}}}$

(36) $y=(1-6x)^{2/3}$

$\frac{dy}{dx}=\frac{2}{3}(1-6x)^{-1/3}(-6)$

$\boxed{\frac{dy}{dx}=\frac{-4}{\sqrt[3]{1-6x}}}$

(38) $y=\frac{x}{\sqrt{x^2+1}}=x(x^2+1)^{-1/2}$

$\frac{dy}{dx}=(x^2+1)^{-1/2}+x\left(-\frac{1}{2}\right)(x^2+1)^{-3/2}(2x)$

$\frac{dy}{dx}=\frac{1}{\sqrt{x^2+1}}-\frac{x^2}{\left(\sqrt{x^2+1}\right)^3}$

$\boxed{\frac{dy}{dx}=\frac{1}{(x^2+1)^{3/2}}}$ (get common denom)

(40) $y=3(2x^{-1/2}+1)^{-1/3}$

$\frac{dy}{dx}=-1(2x^{-1/2}+1)^{-4/3}\cdot\left(-x^{-3/2}\right)$

$\frac{dy}{dx}=\frac{1}{x^{3/2}\left(\frac{2}{\sqrt{x}}+1\right)^{4/3}}\left(\frac{\sqrt{x}^{\,4/3}}{\sqrt{x}^{\,4/3}}\right)$ ← complex fraction

$\frac{dy}{dx}=\frac{(x^{1/2})^{4/3}}{x^{3/2}\left(\sqrt{x}\left(\frac{2}{\sqrt{x}}+1\right)\right)^{4/3}}$

$\frac{dy}{dx}=\frac{x^{2/3}}{x^{3/2}(2+x^{1/2})^{4/3}}$

$\boxed{\frac{dy}{dx}=\frac{1}{x^{5/6}(2+x^{1/2})^{4/3}}}$

(42) $y=\left[\sin(x+5)\right]^{5/4}$

$\frac{dy}{dx}=\frac{5}{4}\left[\sin(x+5)\right]^{1/4}\cdot\cos(x+5)\cdot 1$

$\boxed{\frac{dy}{dx}=\frac{5}{4}\cos(x+5)\sqrt[4]{\sin(x+5)}}$

(43) $f''(x)=x^{-1/3}$

either try all answer choices or work backwards

$f'(x)=\frac{3}{2}x^{2/3}+C_1 \rightarrow \boxed{D}$ ✓

$f(x)=\frac{3}{5}\left(\frac{3}{2}\right)x^{5/3}+C_1x+C_2$

$=\frac{9}{10}x^{5/3}+C_1x+C_2 \rightarrow \boxed{B}$ ✓

~~Not~~ $\boxed{A}$

$f'''(x)=-\frac{1}{3}x^{-4/3} \rightarrow \boxed{C}$ ✓

So $\boxed{\text{B, C, and D}}$

(50) $x^2+xy+y^2=7$

a) tan parallel to x-axis $\rightarrow m=0,\ \frac{dy}{dx}=0$

$\frac{d}{dx}: 2x+y+xy'+2yy'=0$

$y'(x+2y)=-2x-y$

$\boxed{y'=-\frac{2x+y}{2y+x}}$

$=0$ when num $=0$, denom $\neq 0$

$2x+y=0 \rightarrow y=-2x$

plug into eq.

$x^2+x(-2x)+(-2x)^2=7$

$x^2-2x^2+4x^2=7$

$3x^2=7$

$x=\pm\sqrt{\frac{7}{3}},\ y=-2x$

$\boxed{\text{parallel to x-axis @ } \left(\sqrt{\tfrac{7}{3}}, -2\sqrt{\tfrac{7}{3}}\right) \text{ and } \left(-\sqrt{\tfrac{7}{3}}, 2\sqrt{\tfrac{7}{3}}\right)}$

b) parallel to y-axis $\rightarrow \frac{dy}{dx}=\infty$

denom $=0$, num $\neq 0$

$2y+x=0$

$x=-2y$

(plug into eq)

$(-2y)^2+(-2y)y+y^2=7$

$4y^2-2y^2+y^2=7$

$3y^2=7$

$y=\pm\sqrt{\frac{7}{3}},\ x=-2y$

$\boxed{\text{parallel to y-axis @ } \left(-2\sqrt{\tfrac{7}{3}}, \sqrt{\tfrac{7}{3}}\right) \text{ AND } \left(2\sqrt{\tfrac{7}{3}}, -\sqrt{\tfrac{7}{3}}\right)}$

(57) Find normals of graph $xy+2x-y=0$ that are // to $y=-2x$, $\boxed{m=-2}$

$\frac{d}{dx}: y+xy'+2-y'=0$

$y'=\frac{-y-2}{x-1}=\boxed{-\frac{y+2}{x-1}=y'}$

we want normals to be -2

So tangents of curve must be $+\frac{1}{2}=y'$

$y'=-\frac{y+2}{x-1}=\frac{1}{2}$

$y+2=\frac{1}{2}(1-x)$

$y=-\frac{3}{2}-\frac{1}{2}x$

plug into eq

$x\left(-\frac{3}{2}-\frac{1}{2}x\right)+2x+\frac{3}{2}+\frac{1}{2}x=0$

$-\frac{3}{2}x-\frac{1}{2}x^2+2x+\frac{3}{2}+\frac{1}{2}x=0$

$-\frac{1}{2}x^2+x+\frac{3}{2}=0$

$x^2-2x-3=0$

$(x-3)(x+1)=0$

$x=3,\ x=-1$

$\boxed{\text{@ }(-1,-1),\ y=-2x-3 \quad \text{@ }(3,-3),\ y=-2x+3}$

—Koji