Cal ABC § 3.8 p. 170 (6-8, 10-26 even, 27, 28, 31, 35, 36, 41)

—Korpi
period $\sqrt{-1}$

⑥ $y = s\sqrt{1-s^2} + \cos^{-1}s$

$$\frac{dy}{ds} = 1(\sqrt{1-s^2}) + s(\tfrac{1}{2})(1-s^2)^{-1/2}(-2s) - \frac{1}{\sqrt{1-s^2}}$$

$$= \sqrt{1-s^2} - \frac{s^2}{\sqrt{1-s^2}} - \frac{1}{\sqrt{1-s^2}}$$

$$= \frac{1-s^2-s^2-1}{\sqrt{1-s^2}}$$

$$\boxed{\frac{dy}{ds} = \frac{-2s^2}{\sqrt{1-s^2}}}$$

⑦ $y = x\sin^{-1}x + \sqrt{1-x^2}$

$$\frac{dy}{dx} = \sin^{-1}x + \frac{x}{\sqrt{1-x^2}} + \tfrac{1}{2}(1-x^2)^{-1/2}(-2x)$$

$$= \sin^{-1}x + \frac{x}{\sqrt{1-x^2}} - \frac{x}{\sqrt{1-x^2}}$$

$$\boxed{\frac{dy}{dx} = \sin^{-1}x}$$

⑧ $y = \dfrac{1}{\sin^{-1}(2x)} = (\sin^{-1}(2x))^{-1}$

$$\frac{dy}{dx} = -1(\sin^{-1}(2x))^{-2} \cdot \frac{2}{\sqrt{1-4x^2}}$$

$$\boxed{\frac{dy}{dx} = \frac{-2}{(\sin^{-1}(2x))^2\sqrt{1-4x^2}}}$$

⑩ $x(t) = \sin^{-1}(\tfrac{1}{4}t^{1/2})$, $v(4) = ?$

$$x'(t) = v(t) = \frac{1}{\sqrt{1-(\frac{1}{4}t^{1/2})^2}} \cdot \left(\tfrac{1}{8}t^{-1/2}\right) = \frac{1}{\sqrt{1-\frac{1}{16}t}} \cdot \frac{1}{8\sqrt{t}}$$

$$v(4) = \frac{1}{\sqrt{1-\frac{1}{4}}}\left(\frac{1}{8\cdot 2}\right) = \frac{1}{\sqrt{\frac{3}{4}}}\left(\frac{1}{16}\right) = \frac{2}{\sqrt{3}}\left(\frac{1}{16}\right) = \boxed{\frac{1}{8\sqrt{3}} = \frac{\sqrt{3}}{24}}$$

⑫ $x(t) = \tan^{-1}(t^2)$, $v(1) = ?$

$$x'(t) = v(t) = \frac{2t}{1+t^4}, \quad v(1) = \frac{2}{2} = \boxed{1}$$

⑭ $y = \sec^{-1}(5s)$

$$\frac{dy}{ds} = \frac{5}{|5s|\sqrt{25s^2-1}}$$

$$\boxed{\frac{dy}{ds} = \frac{1}{|s|\sqrt{25s^2-1}}}$$

⑯ $y = \csc^{-1}(\tfrac{1}{2}x)$

$$\frac{dy}{dx} = \frac{-1/2}{|\frac{1}{2}x|\sqrt{\frac{1}{4}x^2-1}}\left(\frac{\sqrt{4}}{\sqrt{4}}\right)$$

$$= \boxed{\frac{-2}{|x|\sqrt{x^2-4}}}$$

⑱ $y = \cot^{-1}(t^{1/2})$

$$\frac{dy}{dt} = \frac{-\frac{1}{2}t^{-1/2}}{1+t}$$

$$\boxed{\frac{dy}{dt} = \frac{-1}{2\sqrt{t}(1+t)}}$$

⑳ $y = (s^2-1)^{1/2} - \sec^{-1}s$

$$\frac{dy}{ds} = \tfrac{1}{2}(s^2-1)^{-1/2}(2s) - \frac{1}{|s|\sqrt{s^2-1}}$$

$$= \frac{s}{\sqrt{s^2-1}} - \frac{1}{|s|\sqrt{s^2-1}}$$

$$\boxed{= \frac{s|s|-1}{|s|\sqrt{s^2-1}}}$$

㉒ $y = \cot^{-1}(x^{-1}) - \tan^{-1}x$

$$\frac{dy}{dx} = \frac{(-1)(-x^{-2})}{1+x^{-2}} - \frac{1}{1+x^2}$$

$$= \frac{1}{x^2(1+\frac{1}{x^2})} - \frac{1}{1+x^2}$$

$$= \frac{1}{x^2+1} - \frac{1}{1+x^2}$$

$$= \boxed{0}$$

㉔ $y = \tan^{-1}x$ @ $x=2$

$$\frac{dy}{dx} = \frac{1}{1+x^2}, \quad \left.\frac{dy}{dx}\right|_{x=2} = \frac{1}{5}$$

$(x,y) = (2, \tan^{-1}2)$

tan line

$$y - \tan^{-1}2 = \tfrac{1}{5}(x-2)$$

$$\boxed{y = \tfrac{1}{5}x - \tfrac{2}{5} + \tan^{-1}2}$$

$$\boxed{\text{or } y = \tfrac{1}{5}x + 0.707}$$

㉖ $y = \tan^{-1}(x^2)$ @ $x=1$

$$\frac{dy}{dx} = \frac{2x}{1+x^4}, \quad \left.\frac{dy}{dx}\right|_{x=1} = \frac{2}{2} = 1$$

$(x,y) = (1, \tan^{-1}1) = (1, \frac{\pi}{4})$

tan line

$$y - \tfrac{\pi}{4} = 1(x-1)$$

$$\boxed{y = x - 1 + \tfrac{\pi}{4}}$$

$$\boxed{\text{or } y = x - 0.215}$$

㉗ $y = \tan x$ @ $(\frac{\pi}{4}, 1)$

$y' = \sec^2 x$, $y'(\frac{\pi}{4}) = 2$

a) tan line:

$$y - 1 = 2(x - \tfrac{\pi}{4})$$

$$\boxed{y = 2x - \tfrac{\pi}{2} + 1}$$

b) $y = \tan^{-1}(x)$ @ $(1, \frac{\pi}{4})$

$y' = \dfrac{1}{1+x^2}$, $y'(1) = \frac{1}{2}$

tan line

$$y - \tfrac{\pi}{4} = \tfrac{1}{2}(x-1)$$

$$\boxed{y = \tfrac{1}{2}x - \tfrac{1}{2} + \tfrac{\pi}{4}}$$

*Inverses have Reciprocal slopes at corresponding points!!

㉘ $f(x) = x^5 + 2x^3 + x - 1$

a) $f(1) = 3$

$f'(x) = 5x^4 + 6x^2 + 1$

$f'(1) = 12$

b) $f^{-1}(3) = 1$ (from a)

c) $(f^{-1})'(3) = \dfrac{1}{f'(1)} = \boxed{\dfrac{1}{12}}$

Cal ABC §3.8 cont

(31) $t \geq 0$, $x(t) = \arctan t$

a) $x'(t) = v(t) = \frac{1}{1+t^2} \geq 0 \ \forall t \geq 0$

So velocity is always pos.

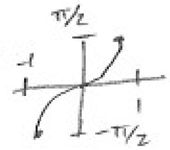
So particle always moves to right.

b) $a(t) = v'(t) = -1(1+t^2)^{-2}(2t) = \frac{-2t}{(1+t^2)^2} < 0, \forall t > 0$

so acceleration is always negative

c) $\lim\limits_{x\to\infty} x(t) = \boxed{\lim\limits_{x\to\infty} \arctan t = \frac{\pi}{2}}$

(35) T or F: $y = \sin^{-1} x$



D: $[-1, 1]$

R: $[-\frac{\pi}{2}, \frac{\pi}{2}]$

True

(36) T or F: $y = \tan^{-1} x$ (see graph at left)

D: $\mathbb{R}$

R: $(-\frac{\pi}{2}, \frac{\pi}{2})$

FALSE

(41) $y = \tan^{-1} x$ (see graph above)

a) $\lim\limits_{x\to\infty} \tan^{-1} x = \pi/2$

b) $\lim\limits_{x\to-\infty} \tan^{-1} x = -\frac{\pi}{2}$

c) horz tans? $\frac{dy}{dx} = 0$?

$\frac{d}{dx}\left[\tan^{-1} x\right] = \frac{1}{1+x^2} \neq 0$

So No horz tans

pg 2/2

– Kenji