Cal ABC §3.9 p. 178 (2–42 even, 43, 44, 47, 49, 51, 57, 58, AP1–4)

Korpi
per. $\sqrt{-1}$

(2) $y = e^{2x}$
$\frac{dy}{dx} = 2e^{2x}$

(4) $y = e^{-5x}$
$\frac{dy}{dx} = -5e^{-5x}$

(6) $y = e^{-x/4}$
$\frac{dy}{dx} = -\frac{1}{4}e^{-x/4}$

(8) $y = x^2e^x - xe^x$
$\frac{dy}{dx} = 2xe^x + x^2e^x - e^x - xe^x$
$= xe^x + x^2e^x - e^x$
$= \boxed{e^x(x^2+x-1)}$

(10) $y = e^{x^2}$
$\frac{dy}{dx} = 2xe^{x^2}$

(12) $y = 9^{-x}$
$\frac{dy}{dx} = -9^{-x}\ln 9$

(14) $y = 3^{\cot x}$
$\frac{dy}{dx} = -\csc^2 x \cdot 3^{\cot x}\ln 3$

(16) $y = (\ln x)^2$
$\frac{dy}{dx} = 2\ln x \cdot \frac{1}{x}$
$= \boxed{\frac{2\ln x}{x} \text{ or } \frac{\ln x^2}{x}}$

(18) $y = \ln\left(\frac{10}{x}\right) = \ln 10 - \ln x$
$\boxed{\frac{dy}{dx} = -\frac{1}{x}}$

(20) $y = x\ln x - x$
$\frac{dy}{dx} = \ln x + \frac{x}{x} - 1$
$= \boxed{\ln x}$

(22) $y = \log_5 x^{1/2}$
$\frac{dy}{dx} = \frac{\frac{1}{2}x^{-1/2}}{(\ln 5)x^{1/2}}$
$= \boxed{\frac{1}{2\ln 5\, x}}$

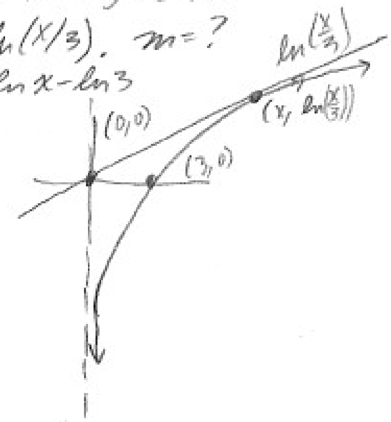
(24) $y = \frac{1}{\log_2 x} = (\log_2 x)^{-1}$
$\frac{dy}{dx} = -1(\log_2 x)^{-2} \cdot \frac{1}{x\ln 2}$
$= \boxed{\frac{-1}{x\ln 2 \cdot (\log_2 x)^2}}$

(26) $y = \log_3(1 + x\ln 3)$
$\frac{dy}{dx} = \frac{\ln 3}{\ln 3(1+x\ln 3)}$
$= \boxed{\frac{1}{1+x\ln 3}}$

(28) $y = \ln 10^x = x \cdot \ln 10$
$\boxed{\frac{dy}{dx} = \ln 10}$

(30) $y = 2e^x - 1$
where is tan line ⊥ to $y = -3x+2$
(where is slope $= \frac{1}{3}$?)
$\frac{dy}{dx} = 2e^x = \frac{1}{3} \rightarrow e^x = \frac{1}{6}$
$x = \ln\frac{1}{6} = \ln 1 - \ln 6$
$x = -\ln 6$
$y = 2e^{\ln 6^{-1}} - 1$
$= 2\left(\frac{1}{6}\right) - 1$
$= \frac{1}{3} - 1 = -\frac{2}{3}$
$\boxed{(x,y) = \left(-\ln 6, -\frac{2}{3}\right) \approx (-1.792, -0.667)}$

(32) line w/ slope m passes through $(0,0)$ and is tangent to $y = \ln(x/3)$. $m = ?$
$y = \ln x - \ln 3$
$\frac{dy}{dx} = \frac{1}{x} = m(x)$
$m = \frac{\Delta y}{\Delta x} = \frac{\ln\left(\frac{x}{3}\right) - 0}{x - 0}$
$= \frac{1}{x}\ln\left(\frac{x}{3}\right) = \frac{dy}{dx} = \frac{1}{x}$
$\ln\left(\frac{x}{3}\right) = 1$
$\frac{x}{3} = e$
$\boxed{x = 3e}$
So $m = \frac{1}{x} = \boxed{\frac{1}{3e} = m}$



(34) $y = x^{1+\sqrt{2}}$
$y' = (1+\sqrt{2})x^{\sqrt{2}}$

(36) $y = x^{1-e}$
$y' = (1-e)x^{-e}$

(38) $f(x) = \ln(2x+2)$
$f'(x) = \frac{2}{2x+2} = \boxed{\frac{1}{x+1}}$
from $f(x)$: $2x+2>0$
$2x > -2$
$D_{f'}: \boxed{x > -1}$

(40) $f(x) = \ln(x^2+1)$
$D_f: x^2+1>0 \rightarrow x \in \mathbb{R}$
$\boxed{f'(x) = \frac{2x}{x^2+1}}$ $D_{f'}: \mathbb{R}$

(42) $f(x) = \log(x+1)^{1/2}$, $x+1>0$, $x>-1$
$f'(x) = \frac{\frac{1}{2}(x+1)^{-1/2}}{(x+1)^{1/2} \cdot \ln 10}$
$\boxed{f'(x) = \frac{1}{2(x+1)\ln 10},\ D_{f'}: x>-1}$

(43) $y = (\sin x)^x \rightarrow \ln y = x\ln\sin x$
$\frac{d}{dx}: \frac{y'}{y} = \ln(\sin x) + x\left(\frac{\cos x}{\sin x}\right)$
$\boxed{y' = [\ln(\sin x) + x\cot x][(\sin x)^x]}$

(44) $y = x^{\tan x}$
$\ln y = \tan x \cdot \ln x$
$\frac{d}{dx}: \frac{y'}{y} = [\sec^2 x]\ln x + \tan x\left(\frac{1}{x}\right)$
$\boxed{y' = \left[\sec^2 x \ln x + \frac{\tan x}{x}\right]\left[x^{\tan x}\right]}$

(47) $y = x^{\ln x} \rightarrow \ln y = (\ln x)(\ln x)$
$\ln y = (\ln x)^2$
$\frac{d}{dx}: \frac{y'}{y} = 2(\ln x)^1 \cdot \left(\frac{1}{x}\right)$
$\boxed{y' = \left[\frac{2\ln x}{x}\right]\left[x^{\ln x}\right]}$

(49) $y = e^x$, a line tangent to curve passing through $(0,0)$

$m = \frac{\Delta y}{\Delta x} = \frac{e^x}{x} = y' = e^x$
$x = 1$, $y'(1) = e^1 = \boxed{e}$
eq: $y - 0 = e(x-0)$
$\boxed{y = ex}$

Cal ABC
§3.9 cont

(51) Rumor Mongoring

$P(t) = \dfrac{300}{1+2^{4-t}}$

a) $P(0) = \dfrac{300}{1+2^4} = \dfrac{300}{17} \approx 17.647$

or **18 students initially heard**

b) $P'(4) = ?$ From calculator home screen

nderiv(300/(1+2^(4-t)),x,4)

≈ 51.896 people per day

c) spreads at max rate at inflection pt (when $P'(x)$ is a max, or $P(x) = 300/2 = 150$) this occurs at **$x = 4$ days**

$P'(4) \approx 52$ students/day (from b))

(57) T or F: $y = 2^x$

$\dfrac{dy}{dx} = 2^x \cdot \ln 2$ False, it's NOT 2^x

(58) T or F: $y = e^{2x}$

$\dfrac{dy}{dx} = 2e^{2x}$, False, it's NOT $2\ln 2 e^{2x}$

(AP1) $x^3 + 2xy = 9$ @ $x=1$, $1+2y=9$

$2y = 8 \rightarrow y = 4$

$\dfrac{d}{dx}$: $3x^2 + 2y + 2xy' = 0$

Plug in (1,4): $3 + 8 + 2y' = 0$

$y' = -\frac{11}{2} \rightarrow$ **E**

(AP2) $y = (\cos(3x-2))^3$

$\dfrac{dy}{dx} = 3(\cos^2(3x-2)) \cdot (-\sin(3x-2))(3)$

$= -9\cos^2(3x-2)\sin(3x-2) \rightarrow$ **A**

(AP3) $y = \sin^{-1}(2x)$

$\dfrac{dy}{dx} = \dfrac{2}{\sqrt{1-4x^2}} \rightarrow$ **C**

(AP4) $xy^2 - x^3y = 6$

a) $\dfrac{dy}{dx}$: $y^2 + 2xyy' - 3x^2y - x^3y' = 0$

$y'(2x - x^3) = 3x^2y - y^2$

$$y' = \frac{3x^2y - y^2}{2x - x^3}$$

b) $x = 1$: $y^2 - y - 6 = 0$

$(y-3)(y+2) = 0$

$y = 3, y = -2$ pts: (1,3), (1,−2)

@ (1,3): $\left.\dfrac{dy}{dx}\right|_{(1,3)} = \dfrac{3(3)-9}{2-1} = 0 = m$; tan line: **$y = 3$**

@ (1,−2): $\left.\dfrac{dy}{dx}\right|_{(1,-2)} = \dfrac{3(-2)-(-2)^2}{2-1} = -10 = m$

tan line: $y + 2 = -10(x-1)$

$y = -10x + 8$

c) vert tan line: denom = 0, num ≠ 0

$\dfrac{dy}{dx} = \infty$

$2x - x^3 = 0$

$x(2 - x^2) = 0$

$x = 0,\ x = -\sqrt{2},\ x = \sqrt{2}$

$x = 0$ does not work in original equation

so **$x = \pm\sqrt{2}$**