AP Calculus § 6.1 p. 327 (2, 4-6, 9, 10, 17, 19, 20, 24, 25-28, 30, 32, 34-40, 49, 51, 52, 59, 60, 64, 68)

Korpi period $\sqrt{-1}$

(2) $\frac{dy}{dx} = \sec x \tan x - e^x$

MUST show this step!!

$\int dy = \int \sec x \tan x - e^x\, dx$

$\boxed{y = \sec x - e^x + C}$

(4) $\frac{dy}{dx} = \frac{1}{x} - \frac{1}{x^2}$

$\int dy = \int x^{-1} - x^{-2}\, dx$

$\boxed{y = \ln x + \frac{1}{x} + C}$

(5) $\frac{dy}{dx} = 5^x \ln 5 + \frac{1}{x^2+1}$

$\int dy = \int 5^x \ln 5 + \frac{1}{x^2+1}\, dx$

$\boxed{y = 5^x + \arctan x + C}$

(6) $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{x}}$

$\int dy = \int \frac{1}{\sqrt{1-x^2}} - x^{-1/2}\, dx$

$\boxed{y = \arcsin x - 2\sqrt{x} + C}$

(9) $\frac{du}{dx} = (\sec^2 x^5)(5x^4)$

$\int du = \int \sec^2(x^5)\cdot(5x^4)\, dx$

$\boxed{u = \tan(x^5) + C}$

(10) $\frac{dy}{du} = 4(\sin u)^3(\cos u)$

$\int dy = \int 4(\sin u)^3(\cos u)\, du$

$y = (\sin u)^4 + C$

$\boxed{y = \sin^4 u + C}$

(17) $\frac{dy}{dt} = \frac{1}{1+t^2} + 2^t \ln 2,\ (0,3)$

$\int dy = \int \frac{1}{1+t^2} + 2^t \ln 2\, dt$

$\boxed{y = \tan^{-1} t + 2^t + C}$

@ (0,3):

$3 = \tan^{-1} 0 + 2^0 + C$

$3 = 0 + 1 + C$

$\boxed{C = 2}$ so

$\boxed{y = \tan^{-1} t + 2^t + 2}$

(19) $\frac{dV}{dt} = 4\sec t \tan t + e^t + 6t,\ (0,5)$

$\int dV = \int 4\sec t \tan t + e^t + 6t\, dt$

$\boxed{V = 4\sec t + e^t + 3t^2 + C}$

@ (0,5)

$5 = 4\sec 0 + e^0 + 3(0^2) + C$

$5 = 4 + 1 + 0 + C$

$\boxed{C = 0}$ so

$\boxed{V = 4\sec t + e^t + 3t^2}$

(20) $\frac{ds}{dt} = t(3t-2),\ (1,0)$

$\int ds = \int 3t^2 - 2t\, dt$

$\boxed{s = t^3 - t^2 + C}$

@ (1,0):

$0 = 1 - 1 + C$

$\boxed{C = 0}$ so

$\boxed{s = t^3 - t^2}$

(24) $G'(s) = \sqrt[3]{\tan s},\ G(0) = 4$

$\boxed{G(s) = \int_0^s \sqrt[3]{\tan t}\, dt + 4}$

(25) $\frac{dy}{dx} = (\sin x)^2$

slopes ≥ 0

x	$m = dy/dx$
0	0
$\pi/4$	1/2
$\pi/2$	1
$\frac{3\pi}{4}$	1/2
π	0

matches with graph (b)

(26) $\frac{dy}{dx} = (\sin x)^3$

x	$m = \frac{dy}{dx}$
0	0
$\frac{\pi}{2}$	1
π	0
$\frac{3\pi}{2}$	-1

matches with graph (c)

(27) $\frac{dy}{dx} = (\cos x)^2$

always ≥ 0

x	m
0	1
$\frac{\pi}{2}$	0
π	1

matches graph (a)

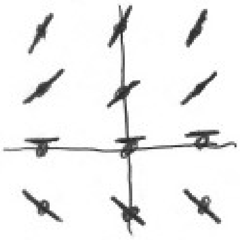
(28) $\frac{dy}{dx} = (\cos x)^3$

x	m
0	1
$\frac{\pi}{2}$	0
π	-1

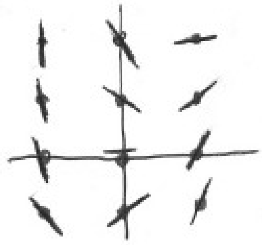
matches graph (d)

AP Cal
S6.1
pg 2.

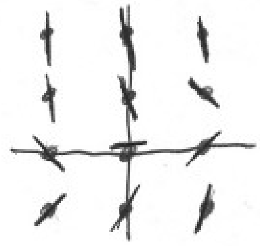
30 $\frac{dy}{dx} = y$



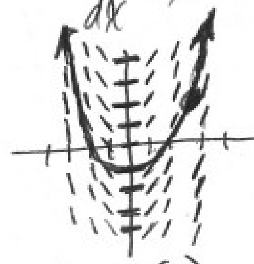
32 $\frac{dy}{dx} = 2x - y$



34 $\frac{dy}{dx} = x - 2y$

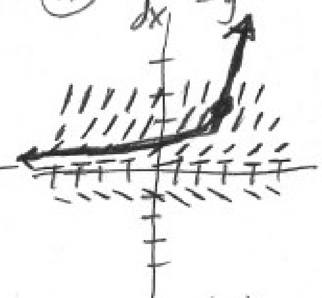


35 $\frac{dy}{dx} = x$



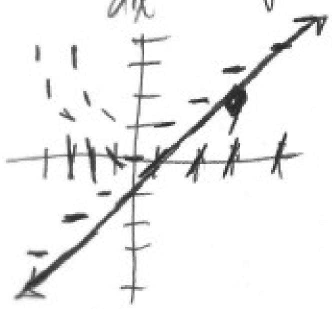
graph (c)

36 $\frac{dy}{dx} = y$



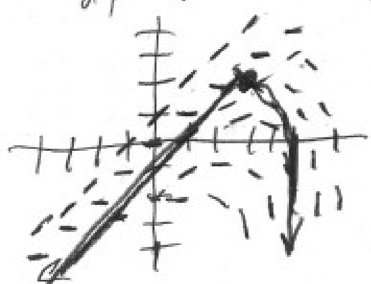
graph (e)

37 $\frac{dy}{dx} = x - y$



graph (a)

38 $\frac{dy}{dx} = y - x$

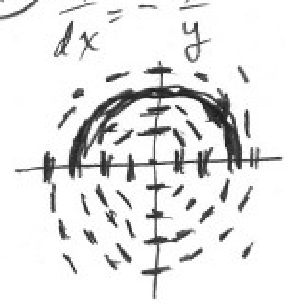


graph (d)

39 $\frac{dy}{dx} = -\frac{y}{x}$

graph (b)

40 $\frac{dy}{dx} = -\frac{x}{y}$



42 *BC only $\frac{dy}{dx} = y - 1$, $(1,3)$, $\Delta x = 0.1$

x	y	$m = \frac{dy}{dx}$	$\Delta y = m\Delta x$	y_{new}
1	3	2	0.2	3.2
1.1	3.2	2.2	0.22	3.42
1.2	3.42	2.42	0.242	3.662
1.3	3.662			

so $\boxed{y(1.3) \approx 3.662}$

44 BC only $\frac{dy}{dx} = 2x - y$, $(1,0)$, $\Delta x = 0.1$

x	y	$m = \frac{dy}{dx}$	$\Delta y = m\Delta x$	y_{new}
1	0	2	0.2	0.2
1.1	0.2	2	0.2	0.4
1.2	0.4	2	0.2	0.6
1.3	0.6			

so $\boxed{y(1.3) \approx 0.6}$

46 BC only $\frac{dy}{dx} = 1 + y$, $(2,0)$, $\Delta x = -0.1$

x	y	$m = \frac{dy}{dx}$	$\Delta y = m\Delta x$	y_{new}
2	0	1	−0.1	−0.1
1.9	−0.1	0.9	−0.09	−0.19
1.8	−0.19	0.81	−0.081	−0.271
1.7	−0.271			

so $\boxed{y(1.7) \approx -0.271}$

48 BC only

$\frac{dy}{dx} = x - 2y$, $(2,1)$, $\Delta x = -0.1$

x	y	$m = \frac{dy}{dx}$	$\Delta y = m\Delta x$	y_{new}
2	1	0	0	1
1.9	1	−0.1	0.01	1.01
1.8	1.01	−0.22	0.022	1.032
1.7	1.032			

so $\boxed{y(1.7) \approx 1.032}$

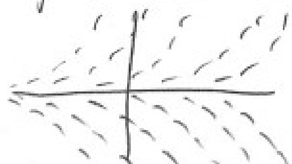
AP Cal
§6.1

(49) $\frac{dy}{dx} = \frac{1}{1+x^2}$, $y(0) = \frac{\pi}{2}$

This is the derivative of Inverse Tangent passing through the point $(0, \frac{\pi}{2})$ so it matches graph (b).

ALSO, $\frac{dy}{dx} > 0 \ \forall x$ so the graph of $y = f(x)$ is <u>monotonic increasing</u>, which rules out graphs (a) and (c)

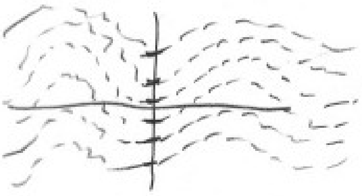
(51) $y = x^2$ cannot be a solution to slopefield



because ① it doesn't look like a parabola

② x^2 has <u>NEG</u>ative slopes in Quadrant II while the graph has <u>POS</u>itive slopes in <u>QII</u>.

(52) $y = \sin x$ cannot be a solution to the slopefield because



① on the y-axis, $\sin x$ is on the sinusoidal axis, and the graph is at a low point ($-\cos x$ graph)

② The slope of $\sin x$ at $x = 0$ is 1, the graph has slopes of ~~zero~~ zero at $x = 0$.

(59) <u>T or F</u>: Any 2 solns to $\frac{dy}{dx} = 5$ are parallel lines:

$\int dy = \int 5\,dx$

$y = 5x + C \rightarrow$ line w/slope of 5

$\boxed{\text{True}}$

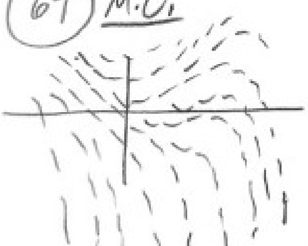
(60) <u>T or F</u>: If $f(x)$ is a soln to $\frac{dy}{dx} = 2x$ then $f^{-1}(x)$ is a soln to $\frac{dy}{dx} = 2y$.

<u>False</u>: for example $f(x) = x^2$ is a soln to $\frac{dy}{dx} = 2x$, but $f^{-1}(x) = \sqrt{x} = y$ is not a soln to $\frac{dy}{dx} = 2y$

since $\frac{dy}{dx} = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}} = \frac{1}{2y}$

$\neq 2y$

(64) <u>M.C.</u>



This slope field matches with $\frac{dy}{dx} = y - |x|$

Choice (A)

(68) a) $y'' = 24x^2 - 10$, $y(1) = 5$
$y'(1) = 3$

$y' = 8x^3 - 10x + C$

$3 = 8 - 10 + C$

$C = 5$, so

$y' = 8x^3 - 10x + 5$

$y = 2x^4 - 5x^2 + 5x + C$

$5 = 2 - 5 + 5 + C$

$C = 3$, so

$\boxed{y = 2x^4 - 5x^2 + 5x + 3}$

b) $y'' = \cos x - \sin x$, $y(0) = 0$

$y' = \sin x + \cos x + C$ $\quad y'(0) = 2$

$2 = 0 + 1 + C$

$C = 1$

$y' = \sin x + \cos x + 1$

$y = -\cos x + \sin x + x + C$

$0 = -1 + 0 + 0 + C$

$C = 1$, so

$\boxed{y = -\cos x + \sin x + x + 1}$

d) $y'' = e^x - x$, $y(0) = 1$
$y'(0) = 0$

$y' = e^x - \frac{1}{2}x^2 + C$

$0 = 1 + C$, $C = -1$

$y' = e^x - \frac{1}{2}x^2 - 1$

$y = e^x - \frac{1}{6}x^3 - x + C$

$1 = 1 + C$, $C = 0$

So $\boxed{y = e^x - \frac{1}{6}x^3 - x}$