APCal §6.2 a) p.337 (4, 10, 12, 19, 22, 26–68 even, 71, 72, 73, 75, 80, 82) — Korpi period $\sqrt{-1}$

(4) $\int \frac{dt}{t^2+1} = \boxed{\arctan t + c}$

(10) $\int 5^x dx = \frac{1}{\ln 5} 5^x + c$

Since $\frac{d}{dx}\left[\frac{1}{\ln 5} 5^x\right] =$

$\frac{1}{\ln 5} \cdot 5^x \cdot 1 \cdot \ln 5 = 5^x$ ✓

(12) $\int \frac{1}{\sqrt{1-u^2}} du = \sin^{-1} u + c$

since $\frac{d}{du}\left[\sin^{-1} u\right] = \frac{1}{\sqrt{1-u^2}}$ ✓

(19) $\int \sec 2x \cdot \tan 2x \, dx$

Let $u = 2x$

$du = 2dx \to dx = \frac{1}{2} du$

So $\int \sec u \cdot \tan u \cdot \frac{1}{2} du$

$= \frac{1}{2} \sec u + C$

$= \boxed{\frac{1}{2} \sec 2x + c}$

(22) $\int \frac{9r^2}{\sqrt{1-r^3}} dr$

Let $u = 1 - r^3$

$du = -3r^2 dr$

$r^2 dr = -\frac{1}{3} du$

So $\int \frac{9}{\sqrt{u}} \left(-\frac{1}{3}\right) du$

$= -3 \int u^{-1/2} du$

$= -3(2) u^{1/2} + c$

$= \boxed{-6(1-r^3)^{1/2} + c}$

(26) $\int \sec^2(x+2) dx$

$u = x + 2$

$du = dx$

So $\int \sec^2 u \, du$

$= \tan u + C$

$= \boxed{\tan(x+2) + c}$

(28) $\int \sec\left(\theta + \frac{\pi}{2}\right) \tan\left(\theta + \frac{\pi}{2}\right) d\theta$

Let $u = \theta + \frac{\pi}{2}$

$du = d\theta$

So $\int \sec u \tan u \, du$

$= \sec u + C$

$= \boxed{\sec\left(\theta + \frac{\pi}{2}\right) + c}$

(30) $\int 3(\sin x)^{-2} dx$

$= 3 \int \csc^2 x \, dx$

$= \boxed{-3 \cot x + C}$

* u-sub doesn't work

(32) $\int \sqrt{\cot x} \cdot \csc^2 x \, dx$

$u = \cot x, \; du = -\csc^2 x \, dx$

$\csc^2 x \, dx = -du$

So $\int u^{1/2} \cdot (-1) du$

$= (-1)\left(\frac{2}{3}\right) u^{3/2} + C$

$= \boxed{-\frac{2}{3} (\cot x)^{3/2} + c}$

(34) $\int \tan^7\left(\frac{x}{2}\right) \sec^2\left(\frac{x}{2}\right) dx$

$\int \left(\tan \frac{x}{2}\right)^7 \cdot \sec^2 \frac{x}{2} \, dx$

Let $u = \tan \frac{x}{2}, \; du = \frac{1}{2} \sec^2 \frac{x}{2} dx$

$\sec^2 \frac{x}{2} dx = 2du$

So $\int u^7 \cdot 2 du$

$= 2\left(\frac{1}{8}\right) u^8 + C$

$= \boxed{\frac{1}{4} \tan^8\left(\frac{x}{2}\right) + c}$

(36) $\int \frac{dx}{\sin^2 3x}$

$= \int \csc^2(3x) dx$

$u = 3x, \; du = 3dx$

$dx = \frac{1}{3} du$

So $\int \csc^2 u \cdot \frac{1}{3} du$

$= -\frac{1}{3} \cot u + C$

$= \boxed{-\frac{1}{3} \cot(3x) + c}$

(38) $\int \frac{6 \cos t}{(2 + \sin t)^2} dt$

$u = 2 + \sin t, \; du = \cos t \, dt$

So $6 \int \frac{1}{u^2} du = 6 \int u^{-2} du$

$= 6(-1) u^{-1} + C$

$= -6(2 + \sin t)^{-1} + C$

$= \boxed{\frac{-6}{2 + \sin t} + c}$

APCal §6.2a) cont

(40) $\int \tan^2 x \sec^2 x\, dx$

$= \int (\tan x)^2 \sec^2 x\, dx$

$u = \tan x,\ du = \sec^2 x\, dx$

So $\int u^2\, du$

$= \frac{1}{3}u^3 + C$

$= \boxed{\frac{1}{3}\tan^3 x + C}$

(42) $\int \frac{40\, dx}{x^2+25}$

$= 40\int \frac{dx}{x^2+25} \cdot \frac{1/25}{1/25}$

$= \frac{40}{25}\int \frac{dx}{\left(\frac{x}{5}\right)^2+1}$

Let $u = \frac{x}{5},\ du = \frac{1}{5}dx$

$dx = 5\, du$

So $\frac{8}{5}\int \frac{1}{u^2+1} \cdot 5\, du$

$= 8\arctan u + C$

$= \boxed{8\arctan\left(\frac{x}{5}\right) + C}$

(44) $\int \frac{dx}{\sqrt{5x+8}}$

$u = 5x+8$

$du = 5dx$

$dx = \frac{1}{5}du$

So $\int \frac{1}{5}\frac{1}{\sqrt{u}}\, du$

$= \frac{1}{5}\int u^{-1/2}\, du$

$= \frac{1}{5}(2)u^{1/2} + C$

$= \boxed{\frac{2}{5}(5x+8)^{1/2} + C}$

(46) $\int \csc x\, dx$

$= \int \csc x\left(\frac{\csc x + \cot x}{\csc x + \cot x}\right)dx$

$= \int \frac{\csc^2 x + \csc x \cot x}{\csc x + \cot x}\, dx$

$u = \csc x + \cot x$

$du = (-\csc x \cot x - \csc^2 x)\, dx$

$(\csc^2 x + \csc x \cot x)\, dx = -du$

So $\int -\frac{1}{u}\, du$

$= -\int u^{-1}\, du$

$= -\ln|u| + C$

$= \boxed{-\ln|\csc x + \cot x| + C}$

(48) $\int \sec^4 x\, dx;\ \sec^2 x = 1 + \tan^2 x$

$\int \sec^2 x(1+\tan^2 x)\, dx$

$\int \sec^2 x + (\tan x)^2 \sec^2 x\, dx$

$= \int \sec^2 x\, dx + \int (\tan x)^2 \sec^2 dx$

$= \boxed{\tan x + \frac{1}{3}\tan^3 x + C}$

(50) $\int 4\cos^2 x\, dx,\ \cos 2x = 1 - 2\cos^2 x$

$2\cos^2 x = 1 - \cos 2x$

$= 2\int (1 - \cos 2x)\, dx$

$= 2\int 1\, dx - 2\int \cos(2x)\, dx$

$= \boxed{2x - \sin 2x + C}$

(52) $\int (\cos^4 x - \sin^4 x)\, dx$

$\cos 2x = \cos^2 - \sin^2 x$

$= \int (\cos^2 x - \sin^2 x)\underbrace{(\cos^2 x + \sin^2 x)}_{=1}\, dx$

$= \int \cos 2x\, dx$

$= \boxed{\frac{1}{2}\sin 2x + C}$

(54) $\int_0^1 r\sqrt{1-r^2}\, dr$

$u = 1 - r^2,\ du = -2r\, dr$

$r\, dr = -\frac{1}{2}du$

when $r=0,\ u=1$

$r=1,\ u=0$

$= \int_1^0 -\frac{1}{2}u^{1/2}$

$= -\frac{1}{2}\left(\frac{2}{3}\right)u^{3/2}\Big|_1^0$

$= -\frac{1}{3}[0-1]$

$= \boxed{\frac{1}{3}}$

§6.2 cal cont.

(56) $\int_{-1}^{1} \frac{5r}{(4+r^2)^2}\,dr$

$u=4+r^2,\ du=2r\,dr$

$r\,dr=\frac{1}{2}du$

$r=-1 \to u=5$

$r=1 \to u=5$

$5\int_5^5 \frac{1}{u^2}\cdot\frac{1}{2}\cdot du$

$=\frac{5}{2}\int_5^5 \frac{1}{u^2}\,du=\boxed{0}$

$\int_a^a f(x)dx=0$

(58) $\int_{-\pi}^{\pi} \frac{\cos x}{\sqrt{4+3\sin x}}\,dx$

$u=4+3\sin x$

$du=3\cos x\,dx$

$\cos x\,dx=\frac{1}{3}du$

$x=-\pi \to u=4$

$x=\pi \to u=4$

$\int_4^4 \frac{1}{3}u^{-1/2}du=\boxed{0}$

$\int_a^a f(x)dx=0$

(60) $\int_0^{\pi/6} \cos^{-3}2\theta\cdot\sin2\theta\,d\theta$

$u=\cos2\theta,\ du=-2\sin2\theta\,d\theta$

$\sin2\theta\,d\theta=-\frac{1}{2}du$

$\theta=0 \to u=1$

$\theta=\pi/6 \to u=\frac{1}{2}$

$-\frac{1}{2}\int_1^{1/2} u^{-3}du$

$=-\frac{1}{2}\left(-\frac{1}{2}\right)u^{-2}\Big|_1^{1/2}$

$=\frac{1}{4}\left[\frac{1}{(1/2)^2}-\frac{1}{1^2}\right]=\frac{1}{4}[4-1]$

$=\boxed{\frac{3}{4}}$

(62) $\int_2^5 \frac{dx}{2x-3}$

$u=2x-3$

$du=2dx$

$dx=\frac{1}{2}du$

$x=2 \to u=1$

$x=5 \to u=7$

$\frac{1}{2}\int_1^7 u^{-1}du$

$=\frac{1}{2}\ln u\Big|_1^7$

$=\frac{1}{2}[\ln 7-\ln 1]$

$=\boxed{\frac{1}{2}\ln 7}$

≈ 0.973

(64) $\int_{\pi/4}^{3\pi/4} \cot x\,dx$

$=\int_{\pi/4}^{3\pi/4} \frac{\cos x}{\sin x}dx$

$u=\sin x,\ du=\cos x\,dx$

$x=\pi/4 \to u=\frac{\sqrt{2}}{2}$

$x=\frac{3\pi}{4} \to u=\frac{\sqrt{2}}{2}$

$\int_{\sqrt{2}/2}^{\sqrt{2}/2} u^{-1}du=\boxed{0}$

$\int_a^a f(x)dx=0$

(66) $\int_0^2 e^x/(3+e^x)\,dx$

$u=3+e^x,\ du=e^x dx$

$x=0 \to u=4$

$x=2 \to u=3+e^2$

$\int_4^{3+e^2} u^{-1}du$

$=\ln|u|\Big|_4^{3+e^2}$

$=\boxed{\ln(3+e^2)-\ln(4)}$

≈ 0.954

(68) $\int_{\pi/6}^{\pi/3} (1-\cos3x)\sin3x\,dx$

$u=1-\cos3x,\ du=3\sin3x\,dx$

$\sin3x\,dx=\frac{1}{3}du$

a) $x=\pi/6 \to u=1$

$x=\pi/3 \to u=2$

$\frac{1}{3}\int_1^2 u\,du$

$=\frac{1}{3}\left(\frac{1}{2}\right)u^2\Big|_1^2$

$=\frac{1}{6}[4-1]=\boxed{\frac{1}{2}}$

b) $=\frac{1}{6}(1-\cos3x)^2\Big|_{\pi/6}^{\pi/3}$

$=\boxed{\frac{1}{2}}$

(80) other page

(71) T or F:

$\int_0^{\pi/4} \tan^3x\sec^2x\,dx$

$=\int_0^{\pi/4} u^3du$?

$u=\tan x,\ du=\sec^2 dx$

$x=0 \to u=\tan 0=0$

$x=\pi/4 \to u=\tan\pi/4=1$

$=\int_0^1 u^3du \to \boxed{\text{False}}$

(72) T or F: $f>0$, diff on $[a,b]$

then $\int_a^b \frac{f'(x)}{f(x)}dx=\ln\left(\frac{f(b)}{f(a)}\right)$?

$u=f(x),\ du=f'(x)dx$

$x=a \to u=f(a)$

$x=b \to u=f(b)$

So $\int_{f(a)}^{f(b)} u^{-1}du$

$=\ln|u|\Big|_{f(a)}^{f(b)}$

$=\ln f(b)-\ln(f(a))$

$=\ln\left(\frac{f(b)}{f(a)}\right)$ $\boxed{\text{True}}$

(73) $\int \tan x\,dx$

$=\int \frac{\sin x}{\cos x}dx$

$=-\ln|\cos x|+C$

$\boxed{D}$

(75) $\int_3^5 f(x-a)dx=7$

$\int_{3-a}^{5-a} f(x)dx=$?

Subtract a (Left a) — added a (Left a)

Same area of 7

$\boxed{B}$

Cal
§6.2
cont

80) $\int 2\sec^2 x \tan x \, dx$

a) Let $u = \tan x$
$du = \sec^2 x \, dx$
So $2\int u \, du$
$= u^2 + C$
$= \boxed{\tan^2 x + C}$

b) Let $u = \sec x$
$du = \sec x \tan x \, dx$
So $2\int u \, du$
$= u^2 + C$
$= \boxed{\sec^2 x + C}$

c) we know by the Pythag ID that
$1 + \tan^2 x = \sec^2 x$
So from b)
$\sec^2 x + C = 1 + \tan^2 x + C$
we can call $1 + C$ "new and improved C"
So $\tan^2 x + C_1 = \sec^2 x + C_2$
for some C_1, C_2.

82) Let $u = \tan^{-1} x$

a) $x = \tan u$, $dx = \sec^2 u \, du$ to show

$$\int \frac{dx}{1+x^2} = \int 1 \, du$$

$$= \int \frac{\sec^2 u \, du}{1 + (\tan u)^2}$$

$$= \int \frac{\sec^2 u}{\sec^2 u} du$$ see 80c)

$$= \int 1 \, du \quad \square$$

b) $\int 1 \, du$
$= u + C$
$= \tan^{-1} x + C$