Name_______________________________

AP Cal Review 4.1-4.4

You should be able to perform problems or explain theories that cover the following subjects:

A. Interpretations and properties of definite integrals.
- Computation of Riemann sums using left, right, and midpoint evaluation points.
- Definite integral as a limit of Riemann sums over equal subdivisions.
- Definite integral interpreted as the change of the quantity over an interval:

$$\int_a^b f(x)dx = F(b) - F(a)$$

- Basic properties of definite integrals.

B. Fundamental Theorem of Calculus
- Use of the Fundamental Theorem to evaluate definite integrals
- Use of Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined.

C. Techniques of antidifferentiation
- Antiderivatives following directly from derivatives of basic functions.
- Antiderivatives by substitution of variables (including change of limits for definite integrals)

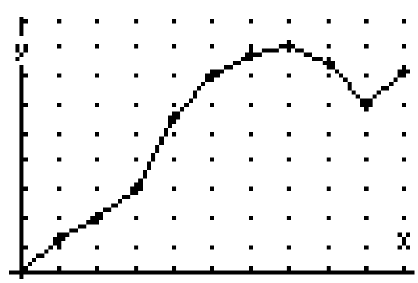
D. Applications of antidifferentiation
- Finding specific antiderivatives using initial conditions, including applications to motion along a line.
- Solving separable differential equations and using them in modeling. In particular, studying the equation $y' = ky$ and exponential growth.

Practice Problems

1. If $\frac{dy}{dx} = xy^2$ and $y(1) = 1$, then $y =$

2. $\int \frac{x^3 + 1}{x^2} dx$

3.



y-axis: Velocity (feet per second), [0,90], y-scl: 10
x-axis: Time (seconds), [0,50], x-scl: 5

t ***(seconds)***	$v(t)$ ***(feet per second)***
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72

The graph of the velocity $v(t)$, in ft/sec, of a car traveling on a straight road, for $0 \le t \le 50$, is shown above. A table of values for $v(t)$, at 5 second intervals of time t, is shown to the right of the graph.

a) During what intervals of time is the acceleration of the car positive? Give a reason for you answer?

b) Find the average acceleration of the car, in $ft/\sec^2$, over the interval $0 \le t \le 50$.

c) Find one approximation for the acceleration of the car, in $ft/\sec^2$, at $t = 40$. Show the computations you used to arrive at your answer.

d) Approximate $\int_0^{50} v(t)dt$ with Riemann sum, using the midpoints of five subintervals of equal length. Using correct units, explain the meaning of the integral.

4. If $\int_a^b f(x)dx = 10$, where $f(x)$ is a continuous function, what would we need to do with the limits of integration to maintain an equivalent area under the transformation $f(x-3)$? How would the area be affected under the transformation $f(x)+5$?

5. Find the values of k for which $\int_{-3}^{k} (x^2 - 5)dx = 0$.

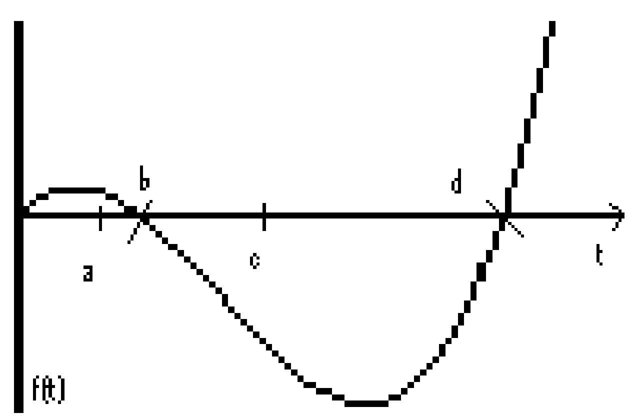
6. $\int_1^4 |4x - 12| dx =$

7. Use the limit definition of the Definite Integral (Riemann Sums) to evaluate the area of the region bounded by $f(x) = x^2 + 2$, the x-axis, and the vertical lines $x = 1$ and $x = 2$.

8. Verify your answer from number 7 using the Fundamental Theorem of Calculus.

9. If $f(x) = g(x) + 7$ for $3 \le x \le 5$, then find $\int_3^5 [f(x) + g(x)]dx$. Leave your answers in terms of $\int_3^5 g(x)dx$.

10. Let f be defined by the graph below, where f is continuous and differentiable on $(0, \infty)$; $f(0) = 0$; $0 < a < b < c < d$. Let $F(x) = \int_a^x f(t)dt$.



a) $F(a) =$

b) $f(b) =$

c) Is $F(b)$ positive, negative, or zero?

d) Is $F(c)$ positive, negative, or zero?

e) Is $F(x)$ increasing or decreasing at $x = c$?

f) Is $F'(a)$ positive, negative, or zero?

g) Is $F(x)$ concave up or concave down at $x = c$?

h) Is $F(x)$ concave up or concave down at $s = d$?

i) At what value of x is $F(x)$ a maximum? A minimum?

j) Is $F(0)$ positive, negative, or zero?

11. $\int_{-4}^{10} g(x)dx = 5$ and $\int_{3}^{10} g(x)dx = -6$, then

a) $\int_{-4}^{10} 2g(x)dx =$ b) $\int_{-4}^{3} g(x)dx =$ c) $\int_{10}^{10} g(x)dx =$

d) $\int_{-4}^{10} (4 + g(x))dx =$ e) $\int_{10}^{-4} g(x)dx =$

12. Assume both f and g are continuous, $a < b$, and $\int_{a}^{b} f(x)dx > \int_{a}^{b} g(x)dx$.

a) Must $\left|\int_{a}^{b} f(x)dx\right| > \left|\int_{a}^{b} g(x)dx\right|$? Why?

b) Must $f(x) > g(x)$ for all x in the interval $[a,b]$?

c) Does it follow that $\int_{a}^{b} |f(x)|dx > \int_{a}^{b} |g(x)|dx$?

13. Assume f is continuous, $a < b$ and $\int_{a}^{b} f(x)dx = 0$.

a) Does it necessarily follow that $f(x) = 0$ for all x in $[a,b]$?

b) Does it necessarily follow that $f(x) = 0$ for at least some x in $[a,b]$?

c) Does it necessarily follow that $\int_{a}^{b} |f(x)|dx = 0$?

d) Does it necessarily follow that $\left|\int_{a}^{b} f(x)dx\right| = 0$?