

Nice Analytic Limit

$$\lim_{x \rightarrow 1} \frac{\sqrt{x} - x^2}{1 - \sqrt{x}}$$

Method 1

$$= \lim_{x \rightarrow 1} \frac{\sqrt{x}(1 - x^{3/2})}{1 - \sqrt{x}}$$

$$= \lim_{x \rightarrow 1} \frac{\sqrt{x}(1 - \sqrt{x}^3)}{1 - \sqrt{x}}$$

this is a difference of cubes:

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

with $a = 1$, $b = \sqrt{x}$

$$= \lim_{x \rightarrow 1} \frac{\sqrt{x}(1 - \sqrt{x})(1 + \sqrt{x} + x)}{(1 - \sqrt{x})}$$

$$= \lim_{x \rightarrow 1} \sqrt{x}(1 + \sqrt{x} + x)$$

$$= 1(1 + 1 + 1)$$

$$= \boxed{3}$$

Method 2 $\lim_{x \rightarrow 1} \frac{\sqrt{x} - x^2}{1 - \sqrt{x}}$

$$= \lim_{x \rightarrow 1} \frac{\sqrt{x} - 1 + 1 - x^2}{1 - \sqrt{x}}$$

*Notice I subtracted then added 1 in the numerator, we'll now group and split up numerators

$$= \lim_{x \rightarrow 1} \frac{(\sqrt{x} - 1)}{1 - \sqrt{x}} + \frac{1 - x^2}{1 - \sqrt{x}}$$

$$= \lim_{x \rightarrow 1} \frac{(-1)(1/\sqrt{x})}{(1/\sqrt{x})} + \frac{(1-x)(1+x)}{1 - \sqrt{x}} \left(\frac{1 + \sqrt{x}}{1 + \sqrt{x}} \right)$$

RATCON ON SECOND TERM

$$= \lim_{x \rightarrow 1} -1 + \frac{(1-x)(1+x)(1+\sqrt{x})}{(1-x)}$$

$$= \lim_{x \rightarrow 1} -1 + (1+x)(1+\sqrt{x})$$

$$= -1 + (1+1)(1+1) = -1 + 4$$

$$= \boxed{3}$$