

BC: MC Practice Questions
TEST 2: Derivatives and Differentiability
NO CALCULATOR

1.

Which of the following defines a function f for which $f(-x) = -f(x)$?

- (A) $f(x) = x^2$ (B) $f(x) = \sin x$ (C) $f(x) = \cos x$
(D) $f(x) = \log x$ (E) $f(x) = e^x$

2.

$\ln(x-2) < 0$ if and only if

- (A) $x < 3$ (B) $0 < x < 3$ (C) $2 < x < 3$
(D) $x > 2$ (E) $x > 3$

3.

If $\begin{cases} f(x) = \frac{\sqrt{2x+5} - \sqrt{x+7}}{x-2}, & \text{for } x \neq 2, \\ f(2) = k \end{cases}$ and if f is continuous at $x = 2$, then $k =$

- (A) 0 (B) $\frac{1}{6}$ (C) $\frac{1}{3}$ (D) 1 (E) $\frac{7}{5}$

4.

If $f(x) = 2 + |x-3|$ for all x , then the value of the derivative $f'(x)$ at $x = 3$ is

- (A) -1 (B) 0 (C) 1 (D) 2 (E) nonexistent

5.

If $f(x) = x^{\frac{1}{3}}(x-2)^{\frac{2}{3}}$ for all x , then the domain of f' is

- (A) $\{x \mid x \neq 0\}$ (B) $\{x \mid x > 0\}$ (C) $\{x \mid 0 \leq x \leq 2\}$
(D) $\{x \mid x \neq 0 \text{ and } x \neq 2\}$ (E) $\{x \mid x \text{ is a real number}\}$

6.

If the solutions of $f(x) = 0$ are -1 and 2 , then the solutions of $f\left(\frac{x}{2}\right) = 0$ are

(A) -1 and 2

(B) $-\frac{1}{2}$ and $\frac{5}{2}$

(C) $-\frac{3}{2}$ and $\frac{3}{2}$

(D) $-\frac{1}{2}$ and 1

(E) -2 and 4

7.

$\lim_{n \rightarrow \infty} \frac{4n^2}{n^2 + 10,000n}$ is

(A) 0

(B) $\frac{1}{2,500}$

(C) 1

(D) 4

(E) nonexistent

8.

If $f(x) = x$, then $f'(5) =$

(A) 0

(B) $\frac{1}{5}$

(C) 1

(D) 5

(E) $\frac{25}{2}$

9.

If $f(x) = e^x$, which of the following is equal to $f'(e)$?

(A) $\lim_{h \rightarrow 0} \frac{e^{x+h}}{h}$

(B) $\lim_{h \rightarrow 0} \frac{e^{x+h} - e^e}{h}$

(C) $\lim_{h \rightarrow 0} \frac{e^{e+h} - e}{h}$

(D) $\lim_{h \rightarrow 0} \frac{e^{x+h} - 1}{h}$

(E) $\lim_{h \rightarrow 0} \frac{e^{e+h} - e^e}{h}$

10.

Which of the following functions are continuous for all real numbers x ?

I. $y = x^{\frac{2}{3}}$

II. $y = e^x$

III. $y = \tan x$

(A) None

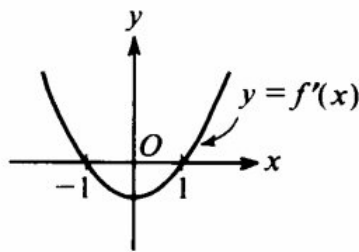
(B) I only

(C) II only

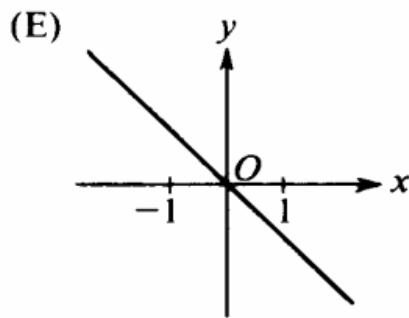
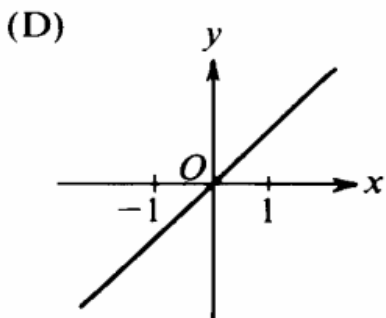
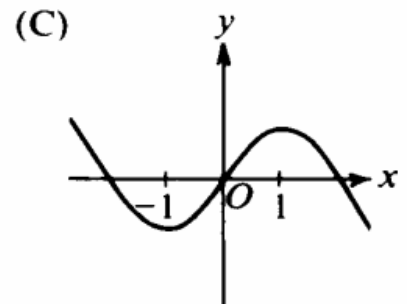
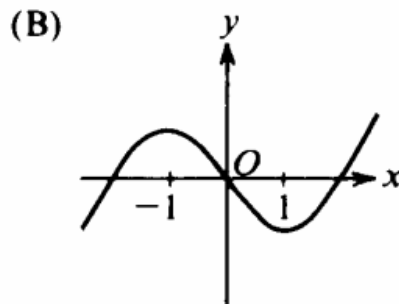
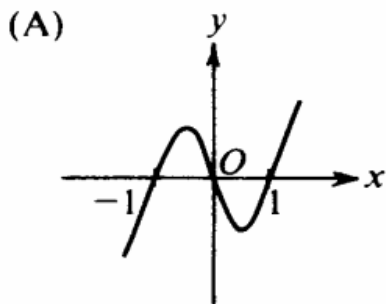
(D) I and II

(E) I and III

11.



The graph of the derivative of f is shown in the figure above. Which of the following could be the graph of f ?



12.

If $\lim_{x \rightarrow a} f(x) = L$, where L is a real number, which of the following must be true?

- (A) $f'(a)$ exists.
- (B) $f(x)$ is continuous at $x = a$.
- (C) $f(x)$ is defined at $x = a$.
- (D) $f(a) = L$
- (E) None of the above

13.

If $f(x) = \sqrt{2x}$, then $f'(2) =$

- (A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{2}}{2}$ (D) 1 (E) $\sqrt{2}$

14.

At $x = 3$, the function given by $f(x) = \begin{cases} x^2 & , \quad x < 3 \\ 6x - 9 & , \quad x \geq 3 \end{cases}$ is

- (A) undefined.
(B) continuous but not differentiable.
(C) differentiable but not continuous.
(D) neither continuous nor differentiable.
(E) both continuous and differentiable.

15.

If $\lim_{x \rightarrow 3} f(x) = 7$, which of the following must be true?

- I. f is continuous at $x = 3$.
II. f is differentiable at $x = 3$.
III. $f(3) = 7$

- (A) None (B) II only (C) III only
(D) I and III only (E) I, II, and III

16.

If the function f is continuous for all real numbers and if $f(x) = \frac{x^2 - 4}{x + 2}$ when $x \neq -2$, then $f(-2) =$

- (A) -4 (B) -2 (C) -1 (D) 0 (E) 2

17.

$$\lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{2 \sin^2 \theta} \text{ is}$$

- (A) 0 (B) $\frac{1}{8}$ (C) $\frac{1}{4}$ (D) 1 (E) nonexistent

18.

If f is a differentiable function, then $f'(a)$ is given by which of the following?

I. $\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$

II. $\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$

III. $\lim_{x \rightarrow a} \frac{f(x+h) - f(x)}{h}$

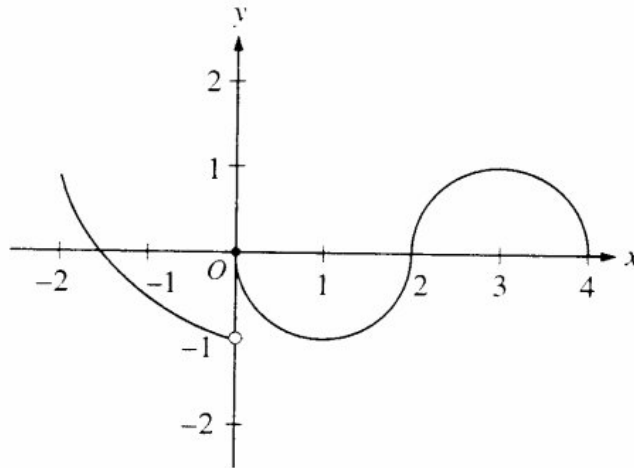
- (A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, and III

19.

$$\lim_{x \rightarrow 1} \frac{x}{\ln x} \text{ is}$$

- (A) 0 (B) $\frac{1}{e}$ (C) 1 (D) e (E) nonexistent

20.



The graph of the function f shown in the figure above has a vertical tangent at the point $(2, 0)$ and horizontal tangents at the points $(1, -1)$ and $(3, 1)$. For what values of x , $-2 < x < 4$, is f not differentiable?

- (A) 0 only (B) 0 and 2 only (C) 1 and 3 only (D) 0, 1, and 3 only (E) 0, 1, 2, and 3

21.

x	0	1	2
$f(x)$	1	k	2

The function f is continuous on the closed interval $[0, 2]$ and has values that are given in the table above. The equation $f(x) = \frac{1}{2}$ must have at least two solutions in the interval $[0, 2]$ if $k =$

- (A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) 3

21. Find the values of a and b such that $f(x) = \begin{cases} 5x + 2, & x < 1 \\ ax^2 + bx, & x \geq 1 \end{cases}$ is differentiable for all x .

- (A) $a = 1, b = 2$ (B) $a = 4, b = -7$ (C) $a = -2, b = 9$
 (D) $a = -5, b = 0$ (E) no such values exist

22.

What is $\lim_{h \rightarrow 0} \frac{8\left(\frac{1}{2} + h\right)^8 - 8\left(\frac{1}{2}\right)^8}{h}$?

- (A) 0 (B) $\frac{1}{2}$ (C) 1 (D) The limit does not exist.
(E) It cannot be determined from the information given.

23.

If $f(x) = x + \sin x$, then $f'(x) =$

- (A) $1 + \cos x$ (B) $1 - \cos x$ (C) $\cos x$
(D) $\sin x - x \cos x$ (E) $\sin x + x \cos x$

24.

The position of a particle moving along a straight line at any time t is given by $s(t) = t^2 + 4t + 4$. What is the acceleration of the particle when $t = 4$?

- (A) 0 (B) 2 (C) 4 (D) 8 (E) 12

25.

If the position of a particle on the x -axis at time t is $-5t^2$, then the average velocity of the particle for $0 \leq t \leq 3$ is

- (A) -45 (B) -30 (C) -15 (D) -10 (E) -5

26.

$\lim_{x \rightarrow 0} (x \csc x)$ is

- (A) $-\infty$ (B) -1 (C) 0 (D) 1 (E) ∞

27.

A particle moves along the x -axis so that at any time $t \geq 0$ its position is given by $x(t) = t^3 - 3t^2 - 9t + 1$. For what values of t is the particle at rest?

- (A) No values (B) 1 only (C) 3 only (D) 5 only (E) 1 and 3

28.

If $f(x) = x^{\frac{3}{2}}$, then $f'(4) =$

- (A) -6 (B) -3 (C) 3 (D) 6 (E) 8

29.

An equation of the line tangent to the graph of $y = \frac{2x+3}{3x-2}$ at the point $(1,5)$ is

- (A) $13x - y = 8$ (B) $13x + y = 18$ (C) $x - 13y = 64$
(D) $x + 13y = 66$ (E) $-2x + 3y = 13$

30.

If $f(x) = (x-1)^2 \sin x$, then $f'(0) =$

- (A) -2 (B) -1 (C) 0 (D) 1 (E) 2

31.

If $f(x) = -x^3 + x + \frac{1}{x}$, then $f'(-1) =$

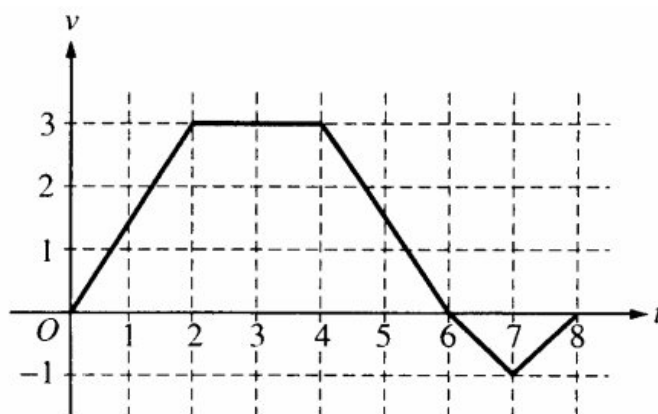
- (A) 3 (B) 1 (C) -1 (D) -3 (E) -5

32.

What is the instantaneous rate of change at $x = 2$ of the function f given by $f(x) = \frac{x^2 - 2}{x - 1}$?

- (A) -2 (B) $\frac{1}{6}$ (C) $\frac{1}{5}$ (D) 2 (E) 6

33.



A bug begins to crawl up a vertical wire at time $t = 0$. The velocity v of the bug at time t , $0 \leq t \leq 8$, is given by the function whose graph is shown above.

At what value of t does the bug change direction?

- (A) 2 (B) 4 (C) 6 (D) 7 (E) 8