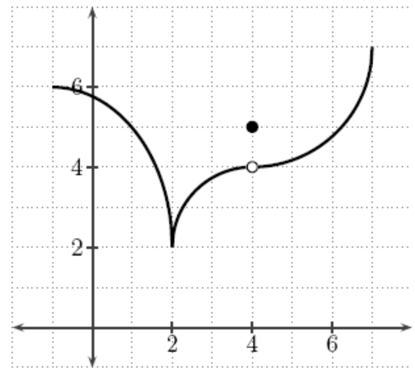
Name____Date____________________ Period________

AP Calculus AB/BC

Practice TEST: Curve Sketch, Optimization, & Related Rates

_____ 1. If *f* is the function whose graph is given at right



Which of the following properties does *f* NOT have?

(A) $\lim_{x\to 4} f(x) = 4$

(B) $f'(x) < 0$ on $(-1,2)$

(C) $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^-} f(x)$

(D) differentiable at $x = 2$

(E) local maximum at $x = 4$

_____ 2. Find the critical values (numbers), x_0, of the function $g(x) = 5x + \sin 5x$ in the interval $(0,\infty)$.

(A) $x_0 = \frac{3n+1}{5}\pi,\ n = 0,1,2,\ldots$ (B) $x_0 = \frac{n}{5}\pi,\ n = 0,1,2,\ldots$ (C) $x_0 = \frac{2n+1}{5}\pi,\ n = 0,1,2,\ldots$

(D) $x_0 = \frac{4n+1}{5}\pi,\ n = 0,1,2,\ldots$ (E) $x_0 = \frac{n+1}{5}\pi,\ n = 0,1,2,\ldots$

_____ 3. Determine the absolute maximum value of $f(x) = \frac{5+2x}{x^2+14}$ on the interval $[-2,4]$.

(A) abs max $= \frac{1}{18}$ (B) abs max $= \frac{13}{30}$ (C) abs max $= \frac{8}{7}$ (D) abs max $= \frac{1}{2}$ (E) none of these

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4. (3 parts) Let *f* be the function defined by $f(x) = x\sqrt{1-x^2} + 2$ on $[-1,1]$.

_____(i) Find the derivative of *f*.

(A) $f'(x) = \frac{1-2x^2}{\sqrt{1-x^2}}$ (B) $f'(x) = \frac{\sqrt{1-x^2}}{2x^2}$ (C) $f'(x) = \frac{2-x^2}{\sqrt{1-x^2}}$

(D) $f'(x) = \sqrt{1-x^2}$ (E) $f'(x) = \frac{2x}{\sqrt{1-x^2}}$ (F) $f'(x) = 2x\sqrt{1-x^2}$

_____ (ii) Find all the critical points of *f* in $(-1,1)$.

(A) $x = \frac{1}{4}$ (B) $x = \pm\frac{1}{4}$ (C) $x = \pm\frac{1}{2}$ (D) $x = \pm\frac{1}{\sqrt{2}}$ (E) $x = \frac{1}{2}$ (F) $x = \frac{1}{\sqrt{2}}$

_____ (iii) Determine the absolute maximum value of *f* $[-1,1]$.

(A) abs. max value $= \frac{7}{2}$ (B) abs. max value $= \frac{5}{2}$ (C) abs. max value $= 1$

(D) abs. max value $=\frac{3}{2}$ (E) abs. max value $=2$ (F) abs. max value $=3$

_____ 5. An advertisement is run to stimulate the sale of cars. After t days, $1\le t\le 48$, the number of cars sold is given by $N(t)=4000+45t^2-t^3$. On what day does the maximum rate of growth sales occur?

(A) on day 17 (B) on day 13 (C) on day 15 (D) on day 16 (E) on day 14

_____ 6. Determine of the function $f(x)=x\sqrt{6-x}$ satisfies the hypothesis of the MVT on the interval $[0,6]$, and if it does, find all numbers c satisfying the conclusion of that theorem.

(A) $c=2,\ 3$ (B) $c=4,\ 5$ (C) $c=5$ (D) $c=3$ (E) $c=4$ (F) hypothesis not satisfied

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_____ 7. Determine if the function $f(x)=x+x^{2/3}(1-x)^{1/3}$ satisfies the hypothesis of the MVT on the interval $[0,1]$. If it does, find all possible values of c that satisfy the conclusion of the MVT.

(A) $c=\frac{3}{4}$ (B) $c=\frac{1}{4}$ (C) $c=\frac{1}{2}$ (D) $c=\frac{1}{3}$ (E) $c=\frac{2}{3}$ (F) hypothesis not satisfied

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_____ 8. As a pre-graduation present, Natalie received a sports car which she drives very fast but very smoothly. She always covers the 53 miles from her home to her favorite stores in Austin in less than 48 minutes. To slow her down, the local DPS officer, Fred Meaney, decides to change the posted speed limit. Which of the speed limits below is the highest he can post, but still catch her speeding at some point on her trip?

(A) Speed Limit = 55 mph (B) Speed Limit = 70 mph (C) Speed Limit = 65 mph
(D) Speed Limit = 50 mph (E) Speed Limit = 60 mph

_____ 9. Determine the increasing and decreasing properties of the function $f(x)=(x-3)^{4/5}(x+1)^{1/5}$ over its domain.

(A) inc: $\left(-1,-\frac{1}{5}\right)$, dec: $\left(-\frac{1}{5},\infty\right)$ (B) inc: $\left(-1,-\frac{1}{5}\right)\cup(3,\infty)$, dec: $\left(-\frac{1}{5},3\right)$

(C) inc: $(-\infty,-1)\cup(3,\infty)$, dec: $(-1,3)$ (D) inc: $\left(-\infty,-\frac{1}{5}\right)\cup(3,\infty)$, dec: $\left(-\frac{1}{5},3\right)$

(E) inc: $\left(-\frac{1}{5},3\right)$, dec: $\left(-1,-\frac{1}{5}\right)\cup(3,\infty)$

_____ 10. Find the values of x at which the graph of $y=x^2-4\cos x$ changes concavity on $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$.

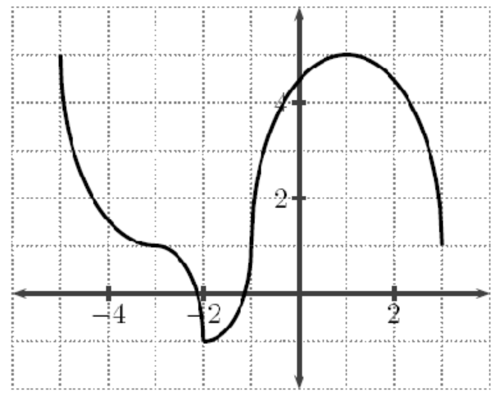
(A) $x=\frac{\pi}{6}$ (B) $x=-\frac{\pi}{3}$ (C) there are no values of x (D) $x=-\frac{\pi}{3},\ \frac{\pi}{3}$

(E) $x=\frac{\pi}{3}$ (F) $x=-\frac{\pi}{6},\ \frac{\pi}{6}$ (G) $x=-\frac{\pi}{6}$

_____ 11. The derivative of a function *f* is given for all *x* by $f'(x)=\left(2x^2+4x-16\right)\left(1+g(x)^2\right)$ where *g* is some unspecified function. At which point(s) will *f* have a local maximum?

(A) local maximum at $x=-4$ (B) local maximum at $x=4$ (C) local maximum at $x=-2$
(D) local maximum at $x=-2$ (E) local maximum at $x=-4,\ 2$

_____ 12. If *f* is a continuous function on $(-5,3)$ whose graph is at right, which of the following properties are satisfied?



I. $f''(x)>0$ on $(-2,1)$
II. *f* has exactly 2 local extrema
III. *f* has exactly 4 critical points.

(A) all of them (B) B only (C) none of them
(D) A and C only (E) C only (F) B and C only
(G) A only

_____ 13. Suppose *g* is the inverse function of a differentiable function *f* and $G(x)=\dfrac{1}{g(x)}$. If $f(3)=7$ and $f'(3)=\dfrac{1}{9}$, find $G'(7)$.

(A) $G'(7)=-5$ (B) $G'(7)=4$ (C) $G'(7)=6$ (D) $G'(7)=-1$ (E) $G'(7)=-4$

_____ 14. On $(-1,1)$ the function $f(x)=6+x^2+\tan\left(\dfrac{\pi x}{2}\right)$ has an inverse *g*. Find the value of $g'(6)$.

(Hint: find the value of $f(0)$.)

(A) $g'(6)=\dfrac{2}{\pi}$ (B) $g'(6)=1$ (C) $g'(6)=\dfrac{\pi}{6}$ (D) $g'(6)=\dfrac{\pi}{2}$ (E) $g'(6)=\dfrac{6}{\pi}$

_____ 15. Determine the derivative of $f(x)=5\sin^{-1}(x/2)$

(A) $f'(x)=\dfrac{2}{\sqrt{1-x^2}}$ (B) $f'(x)=\dfrac{2}{\sqrt{4-x^2}}$ (C) $f'(x)=\dfrac{5}{\sqrt{1-x^2}}$
(D) $f'(x)=\dfrac{5}{\sqrt{4-x^2}}$ (E) $f'(x)=\dfrac{10}{\sqrt{1-x^2}}$ (F) $f'(x)=\dfrac{10}{\sqrt{4-x^2}}$

_____ 16. Find the derivative of $f(x)=\cos\left(\arctan\dfrac{x}{\sqrt{6}}\right)$

(A) $f'(x)=\dfrac{\sqrt{6}x}{\left(x^2+6\right)^{3/2}}$ (B) $f'(x)=\dfrac{x}{\left(x^2+6\right)^{3/2}}$ (C) $f'(x)=-\dfrac{\sqrt{6}x}{\left(x^2+6\right)^{1/2}}$

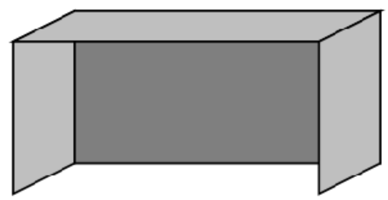
(D) $f'(x)=-\dfrac{x}{\left(x^2+6\right)^{3/2}}$ (E) $f'(x)=-\dfrac{\sqrt{6}x}{\left(x^2+6\right)^{3/2}}$ (F) $f'(x)=\dfrac{\sqrt{6}x}{\left(x^2+6\right)^{1/2}}$

_____ 17. Find the derivative of *f* when $f(x)=3\tan^{-1}x-2\ln\sqrt{\dfrac{1+x}{1-x}}$.

(A) $f'(x)=\dfrac{1+5x^2}{1-x^4}$ (B) $f'(x)=\dfrac{5-x^2}{1-x^2}$ (C) $f'(x)=\dfrac{5+x^2}{1-x^4}$

(D) $f'(x)=\dfrac{1-5x^2}{1-x^2}$ (E) $f'(x)=\dfrac{1-5x^2}{1-x^4}$ (F) $f'(x)=\dfrac{5-x^2}{1-x^4}$

_____ 18. A canvas wind shelter like the one at right is to be built for use along parts of the Guadalupe River. It is to have a back, two square sides, and a top. If $\dfrac{147}{2}$ square feet of canvas is to be used in the construction, find the depth of the shelter for which the space inside is maximized assuming all the canvas is used.



(A) depth $=\dfrac{7}{2}$ feet (B) depth $=\dfrac{7}{4}$ feet (C) depth $=4$ feet (D) depth $=7$ feet (E) none of these

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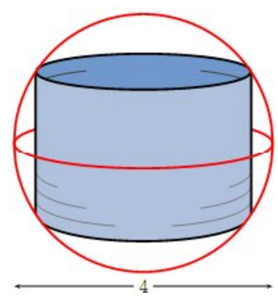
_____ 19. A right circular cylinder is inscribed in a sphere with diameter 4cm as shown. If the cylinder is open at both ends, find the largest possible surface area of the cylinder.



(A) $A=8\text{ cm}^2$ (B) $A=16\text{ cm}^2$ (C) $A=16\pi\text{ cm}^2$

(D) $A=2\text{ cm}^2$ (E) $A=8\pi\text{ cm}^2$ (F) $A=4\pi\text{ cm}^2$

(G) $A=4\text{ cm}^2$ (H) $A=2\pi\text{ cm}^2$ (I) $A=2\text{ cm}^2$

_____ 20. Find an equation for the tangent line to the graph of $f(x)=\dfrac{2}{3}-2x-x^2-\dfrac{1}{3}x^3$ having maximum slope.

(A) $y=x+1$ (B) $y=x-1$ (C) $y+x=1$ (D) $y+x+1=0$ (E) none of these

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_____ 21. If the radius and the height of a right circular cone both increase at a constant rate of $\frac{1}{2}$ centimeters per second, at what rate, in cubic centimeters per second, is the volume increasing when the height is 9 centimeters and the radius is 6 centimeters?

(A) $\frac{\pi}{2}$ (B) 10π (C) 24π (D) 54π (E) 108π

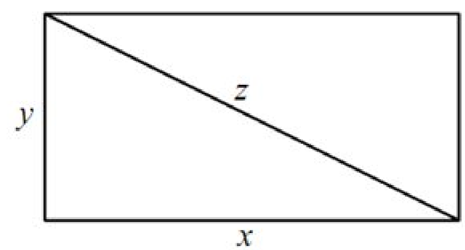
_____ 22. An equation of the tangent line to $y = x^3 + 3x^2 + 2$ at its point of inflection is

(A) $y = -6x - 6$ (B) $y = -3x + 1$ (C) $y = 2x + 10$ (D) $y = 3x - 1$ (E) $y = 4x + 1$

_____ 23. The area of a circular region is increasing at a rate of 96π square meters per second. When the area of the region is 64π square meters, how fast, in meters per second, is the radius of the region increasing?

(A) 6 (B) 8 (C) 16 (D) $4\sqrt{3}$ (E) $12\sqrt{3}$

_____ 24. The sides of the rectangle at right increase in such a way that $\frac{dz}{dt} = 1$ and $\frac{dx}{dt} = 3\frac{dt}{dt}$. At the instant when $x = 4$ and $y = 3$, what is the value of $\frac{dx}{dt}$?



_____ 25. For what value of k will $x + \frac{k}{x}$ have relative maximum at $x = -2$?

(A) -4 (B) -2 (C) 2 (D) 4 (E) None of these

_____ 26. The point on the curve $x^2 + 2y = 0$ that is nearest the point $\left(0, -\frac{1}{2}\right)$ occurs where y is

(A) $\frac{1}{2}$ (B) 0 (C) $-\frac{1}{2}$ (D) -1 (E) None of these

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_____ 27. A point moves on the x-axis in such a way that its velocity at time t $(t > 0)$ is given by $v = \frac{\ln t}{t}$. At what value of t does v attain its maximum?

(A) 1 (B) $\sqrt{e}$ (C) e (D) $\sqrt{e^3}$ (E) There is no maximum value for v.

_____28. At $x=0$, which of the following is true of the function f defined by $f(x)=x^2+e^{-2x}$?

(A) f is increasing (B) f is decreasing (C) f is discontinuous
(D) f has a relative minimum (E) f has a relative maximum

_____ 29. If a function f is continuous for all x and if f has a relative maximum at $(-1,4)$ and a relative minimum at $(3,-2)$, which of the following statements must be true?

(A) The graph of f has a point of inflection somewhere between $x=-1$ and $x=3$
(B) $f'(-1)=0$
(C) The graph of f has a horizontal asymptote
(D) The graph of f has a horizontal tangent line at $x=3$
(E) The graph of f intersects both axes

_____ 30. What are the coordinates of the inflection point on the graph of $y=(x+1)\arctan x$?

(A) $(-1,0)$ (B) $(0,0)$ (C) $(0,1)$ (D) $\left(1,\frac{\pi}{4}\right)$ (E) $\left(1,\frac{\pi}{2}\right)$

_____ 31. The Mean Value Theorem guarantees the existence of a special point on the graph of $y=\sqrt{x}$ between $(0,0)$ and $(4,2)$. What are the coordinates of this point?

(A) $(2,1)$ (B) $(1,1)$ (C) $(2,\sqrt{2})$ (D) $\left(\frac{1}{2},\frac{1}{\sqrt{2}}\right)$ (E) None of these

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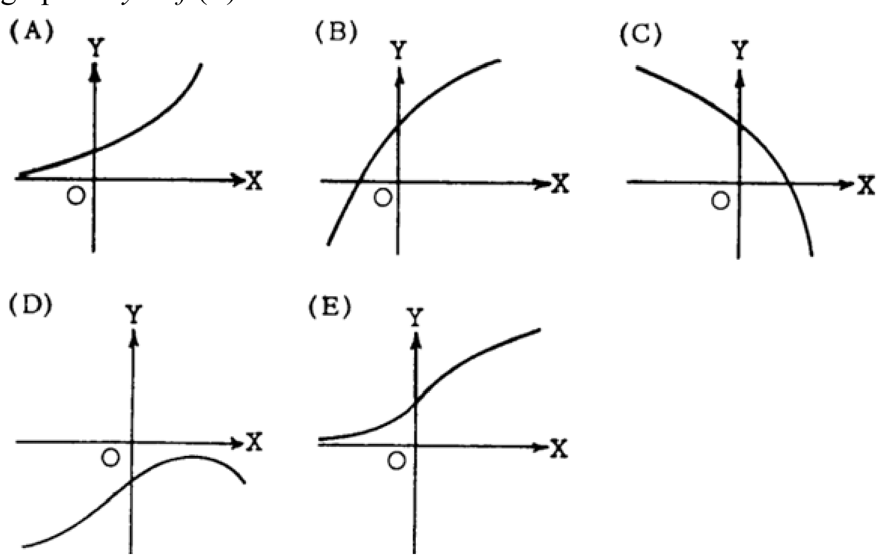
_____ 32. If y is a function of x such that $y'>0$ for all x and $y''<0$ for all x, which of the following could be part of the graph of $y=f(x)$?



_____ 33. The *derivative* of $f(x)=\frac{x^4}{3}-\frac{x^5}{5}$ attains its maximum value at $x=$

(A) -1 B) 0 (C) 1 (D) $\frac{4}{3}$ $\frac{5}{3}$

_____ 34. Given the function defined by $f(x)=3x^5-20x^3$, find all values of x for which the graph of f is concave up.

(A) $x>0$ (B) $-\sqrt{2}<x<0$ or $x>\sqrt{2}$ (C) $-2<x<0$ (D) $x>\sqrt{2}$ (E) $-2<x<2$

_____ 35. If $f(x)=x+\frac{1}{x}$, then the set of values for which f increases is

(A) $(-\infty,-1]\cup[1,\infty)$ (B) $[-1,1]$ (C) $(-\infty,\infty)$ (D) $(0,\infty)$ (E) $(-\infty,0)\cup(0,\infty)$

_____ 36. Let g be a continuous function on the closed interval $[0,1]$. Let $g(0)=1$ and $g(1)=0$. Which of the following is NOT necessarily true?

(A) $\exists h\in[0,1]\ni g(h)\geq g(x)\forall x\in[0,1]$

(B) $\forall a,b\in[0,1]$, if $a=b$, then $g(a)=g(b)$.

(C) $\exists h\in[0,1]\ni g(h)=\frac{1}{2}$

(D) $\exists h\in[0.1]\ni g(h)=\frac{3}{2}$

(E) $\forall h\in(0,1)$, $\lim_{x\to h}g(x)=g(h)$

_____ 37. If $f(x)=\frac{1}{3}x^3-4x^2+12x-5$ and the domain is the set of all x such that $0\leq x\leq 9$, then the absolute maximum value of the function f occurs when x is

(A) 0 (B) 2 (C) 4 (D) 6 (E) 9

_____ 38. If f is a continuous function defined for all real numbers x and if the maximum value of $f(x)$ is 5 and the minimum value of $f(x)$ is -7, then which of the following must be true?

I. The maximum value of $f(|x|)$ is 5.

II. The maximum value of $|f(x)|$ is 7.

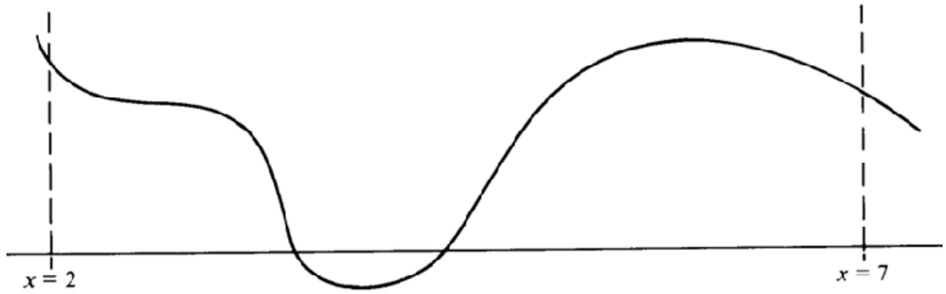
III. The minimum value of $f(|x|)$ is 0.

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, and III

_____ 39. Let f be the function given by $f(x) = x^3 - 3x^2$. What are the values of c that satisfy the conclusion of the Mean Value Theorem of differential calculus on the closed interval $[0,3]$?

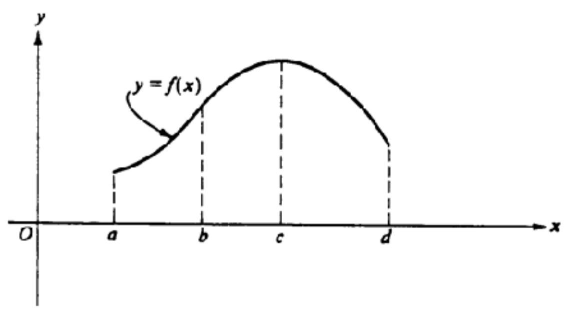
(A) 0 only (B) 2 only (C) 3 only (D) 0 and 3 (E) 2 and 3

_____ 40. The graph of $y = f(x)$ on the closed interval $[2,7]$ is shown below. How many points of inflection does this graph have on this interval?



(A) one (B) two (C) three (D) four (E) five

_____ 41. The graph of $y = f(x)$ is shown at right. On which of the following intervals are $\frac{dy}{dx} > 0$ and $\frac{d^2y}{dx^2} < 0$?



I. $a < x < b$
II. $b < x < c$
III. $c < x < d$

(A) I only (B) II only (C) III only (D) I and II (E) II and III

_____ 42. A street light is on top of a 10 foot pole. Neil, who is 6 feet tall, walks away from the pole at a rate of 4 feet per second. At what speed is the tip of Neil's shadow moving from the base of the pole when he is 10 feet from the pole?

(A) 9 ft/sec (B) 10 ft/sec (C) 11 ft/sec (D) 12 ft/sec (E) 8 ft/sec

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_____ 43. A baseball diamond is a square with side 90 feet. If a batter hits the ball and runs towards first base with a speed of 25 ft/sec, at what speed is his distance from second base decreasing when he is two thirds of the way to first base?

(A) $\frac{5}{2}\sqrt{10}$ ft/sec (B) $\frac{3}{2}\sqrt{10}$ ft/sec (C) $4\sqrt{5}$ ft/sec (D) $2\sqrt{10}$ ft/sec (E) $3\sqrt{5}$ ft/sec

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_____ 44. A point is moving on the graph of $5x^3 + 6y^3 = xy$. When the point is at $\left(\frac{1}{11}, \frac{1}{11}\right)$, its y-coordinate is increasing at a speed of 5 units per second. What is the speed of the x-coordinate at that time and in which direction is the x-coordinate moving?

(A) 8 units/sec, increasing x (B) $\frac{17}{2}$ units/sec, decreasing x (C) $\frac{17}{2}$ units/sec, increasing x

(D) $\frac{33}{4}$ units/sec, decreasing x (E) 8 units/sec, decreasing x (F) $\frac{35}{4}$ units/sec, decreasing x

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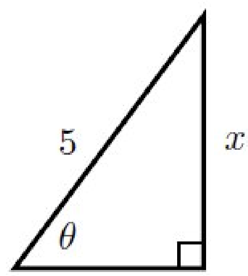
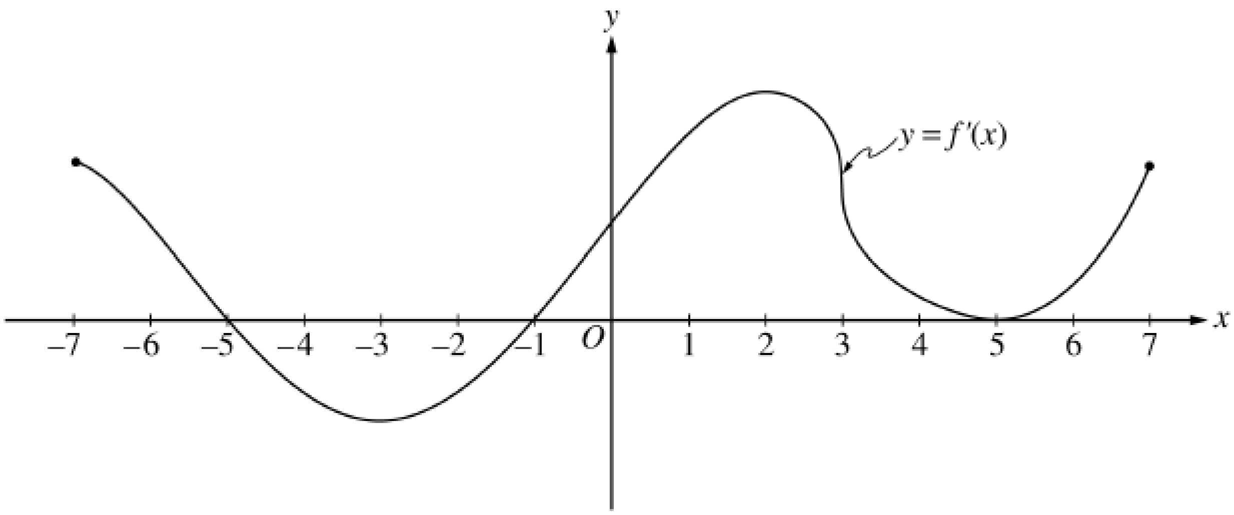
_____ 45. In the right-triangle shown at right, the angle θ is increasing at a constant rate of 2 radians per hour. At what rate is the side length of x increasing when $x = 4$ feet?



(A) 8 ft/hour (B) 4 ft/hour (C) 10 ft/hour (D) 6 ft/hour (E) 2 ft/hour

Part II: Free Response

1. 2000 AB3



The figure above shows the graph of f', the derivative of the function f, for $-7 \le x \le 7$. The graph of f' has horizontal tangent lines at $x = -3$, $x = 2$, and $x = 5$, and a vertical tangent line at $x = 3$.

(a) Find all values of x, for $-7 < x < 7$, at which f attains a relative minimum. Justify your answer.

(b) Find all values of x, for $-7 < x < 7$, at which f attains a relative maximum. Justify your answer.

(c) Find all values of x, for $-7 < x < 7$, at which $f''(x) < 0$.

(d) At what value of x, for $-7 \le x \le 7$, does f attain its absolute maximum? Justify your answer.

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2. Consider the curve given by $xy^2 - x^3y = 6$

(a) Show that $\dfrac{dy}{dx} = \dfrac{3x^2y - y^2}{2xy - x^3}$

(b) Find all points (x, y) on the curve whose x-coordinate is 1, and write an equation for the tangent line at each of these points.

(c) Find the x-coordinate of each point on the curve where the tangent line is vertical.

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3. 2001 AB4

Let h be a function defined for all $x \ne 0$ such that $h(4) = -3$ and the derivative of h is given by

$h'(x) = \dfrac{x^2 - 2}{x}$ for all $x \ne 0$.

(a) Find all values of x for which the graph of h has a horizontal tangent, and determine whether h has a local maximum, a local minimum, or neither at each of these values. Justify your answers.

(b) On what intervals, if any, is the graph of h concave up? Justify your answer.

(c) Write an equation for the line tangent to the graph of h at $x = 4$.

(d) Does the line tangent to the graph of h at $x = 4$ lie above or below the graph of h for $x > 4$? Why?

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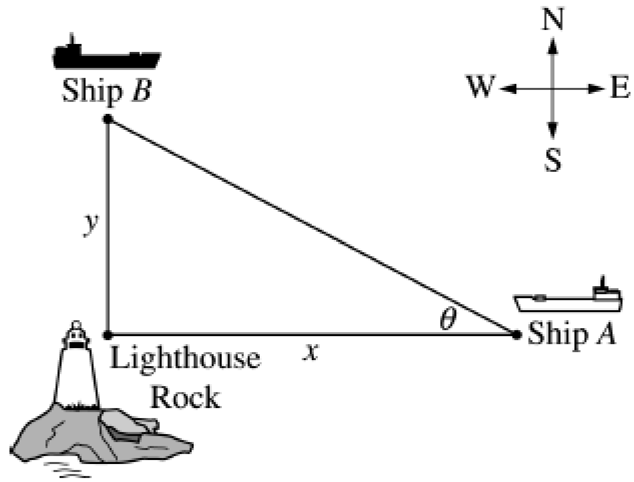
4. 2002 AB6 form B (related rates)

Ship A is traveling due west toward Lighthouse Rock at a speed of 15 kilometers per hour (km/hr). Ship B is traveling due north away from Lighthouse Rock at a speed of 10 km/hr. Let x be the distance between Ship A and Lighthouse Rock at time t, and let y be the distance between Ship B and Lighthouse Rock at time t, as shown in the figure at right.



(a) Find the distance, in kilometers, between Ship A and Ship B when $x = 4$ km and $y = 3$ km.

(b) Find the rate of change, in km/hr, of the distance between the two ships when $x = 4$ km and $y = 3$ km.

(c) Let θ be the angle shown in the figure. Find the rate of change of θ, in radians per hour, when $x = 4$ km and $y = 3$ km.

5. 1999 AB4

Suppose that the function f has a continuous second derivative for all x, and that $f(0) = 2$, $f'(0) = -3$, and $f''(0) = 0$. Let g be a function whose derivative is given by $g'(x) = e^{-2x}(3f(x) + 2f'(x))$ for all x.

(a) Write an equation of the line tangent to the graph of f at the point where $x = 0$.

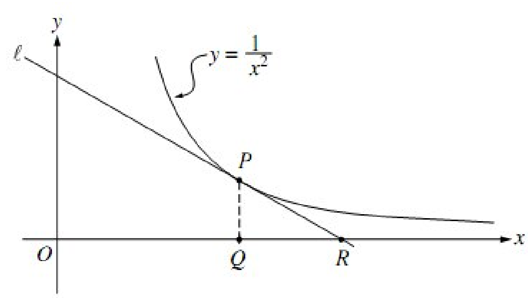
(b) Is there sufficient information to determine whether or not the graph of f has a point of inflection when $x = 0$? Explain your answer.

(c) Given that $g(0) = 4$, write an equation of the line tangent to the graph of g at the point where $x = 0$.

(d) Show that $g''(x) = e^{-2x}(-6f(x) - f'(x) + 2f''(x))$. Does g have a local maximum at $x = 0$? Justify your answer.

6. 1999 AB6 Related Rate

In the figure above, line ℓ is tangent to the graph of $y = \dfrac{1}{x^2}$ at point P, with coordinates $\left(w, \dfrac{1}{w^2}\right)$, where $w > 0$. Point Q has coordinates $(w, 0)$. Line ℓ crosses the x–axis at the point R, with coordinates $(k, 0)$.



(a) Find the value of k when $w = 3$.

(b) For all $w > 0$, find k in terms of w.

(c) Suppose that w is increasing at the constant rate of 7 units per second. When $w = 5$, what is the rate of change of k with respect to time?

(d) Suppose that w is increasing at the constant rate of 7 units per second. When $w = 5$, what is the rate of change of the area of ΔPQR with respect to time? Determine whether the area is increasing or decreasing at this instant.

7. 1998 AB2 (Calculator)

Let f be the function given by $f(x) = 2xe^{2x}$.

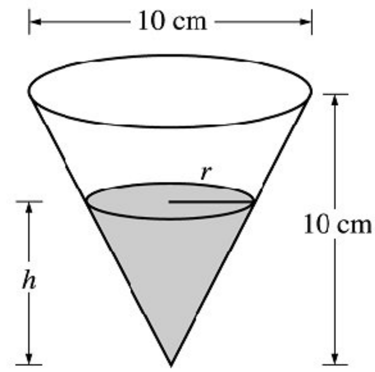
(a) Find $\lim_{x \to -\infty} f(x)$ and $\lim_{x \to \infty} f(x)$.

(b) Find the absolute minimum value of f. Justify that your answer is an absolute minimum.

(c) What is the range of f?

(d) Consider the family of functions defined by $y = bxe^{bx}$, where b is a nonzero constant. Show that the absolute minimum value of bxe^{bx} is the same for all nonzero values of b.

8. 2002 AB5



A container has the shape of an open right circular cone, as shown in the figure above. The height of the container is 10 cm and the diameter of the opening is 10 cm. Water in the container is evaporating so that its depth h is changing at the constant rate of $\frac{-3}{10}$ cm/hr.

(The volume of a cone of height h and radius r is given by $V = \frac{1}{3}\pi r^2 h$.)

(a) Find the volume V of water in the container when $h = 5$ cm. Indicate units of measure.

(b) Find the rate of change of the volume of water in the container, with respect to time, when $h = 5$ cm. Indicate units of measure.

(c) Show that the rate of change of the volume of water in the container due to evaporation is directly proportional to the exposed surface area of the water. What is the constant of proportionality?