Name_____ Date_____ Period_____

BC Calculus Practice Test: All Rules of Differentiation

Part I: Multiple Choice

1.

Let f and g be functions that are differentiable everywhere. If g is the inverse function of f and if $g(-2)=5$ and $f'(5)=-\frac{1}{2}$, then $g'(-2)=$

(A) 2 (B) $\frac{1}{2}$ (C) $\frac{1}{5}$ (D) $-\frac{1}{5}$ (E) -2

2.

If $\frac{dy}{dx}=\sqrt{1-y^2}$, then $\frac{d^2y}{dx^2}=$

(A) $-2y$ (B) $-y$ (C) $\frac{-y}{\sqrt{1-y^2}}$ (D) y (E) $\frac{1}{2}$

3.

The slope of the line tangent to the curve $y^2+(xy+1)^3=0$ at $(2,-1)$ is

(A) $-\frac{3}{2}$ (B) $-\frac{3}{4}$ (C) 0 (D) $\frac{3}{4}$ (E) $\frac{3}{2}$

4.

If $f(x)=\tan(2x)$, then $f'\left(\frac{\pi}{6}\right)=$

(A) $\sqrt{3}$ (B) $2\sqrt{3}$ (C) 4 (D) $4\sqrt{3}$ (E) 8

5.

What is the instantaneous rate of change at $x=2$ of the function f given by $f(x)=\frac{x^2-2}{x-1}$?

(A) -2 (B) $\frac{1}{6}$ (C) $\frac{1}{2}$ (D) 2 (E) 6

6.

If f and g are twice differentiable and if $h(x) = f\big(g(x)\big)$, then $h''(x) =$

(A) $f''\big(g(x)\big)\big[g'(x)\big]^2 + f'\big(g(x)\big)g''(x)$

(B) $f''\big(g(x)\big)g'(x) + f'\big(g(x)\big)g''(x)$

(C) $f''\big(g(x)\big)\big[g'(x)\big]^2$

(D) $f''\big(g(x)\big)g''(x)$

(E) $f''\big(g(x)\big)$

7.

If $x^2 + xy = 10$, then when $x = 2$, $\dfrac{dy}{dx} =$

(A) $-\dfrac{7}{2}$ (B) -2 (C) $\dfrac{2}{7}$ (D) $\dfrac{3}{2}$ (E) $\dfrac{7}{2}$

8.

Let f be a continuous function on the closed interval $[-3, 6]$. If $f(-3) = -1$ and $f(6) = 3$, then the Intermediate Value Theorem guarantees that

(A) $f(0) = 0$

(B) $f'(c) = \dfrac{4}{9}$ for at least one c between -3 and 6

(C) $-1 \le f(x) \le 3$ for all x between -3 and 6

(D) $f(c) = 1$ for at least one c between -3 and 6

(E) $f(c) = 0$ for at least one c between -1 and 3

9.

The line normal to the curve $y = \sqrt{16 - x}$ at the point $(0, 4)$ has slope

(A) 8 (B) 4 (C) $\dfrac{1}{8}$ (D) $-\dfrac{1}{8}$ (E) -8

10. (Calculator Active)

The position of an object attached to a spring is given by $y(t)=\frac{1}{6}\cos(5t)-\frac{1}{4}\sin(5t)$, where t is time in seconds. In the first 4 seconds, how many times is the velocity of the object equal to 0?

(A) Zero
(B) Three
(C) Five
(D) Six
(E) Seven

11.

Let $f(x)=\sqrt{x}$. If the rate of change of f at $x=c$ is twice its rate of change at $x=1$, then $c=$

(A) $\frac{1}{4}$ (B) 1 (C) 4 (D) $\frac{1}{\sqrt{2}}$ (E) $\frac{1}{2\sqrt{2}}$

12.

If $x^2+y^2=25$, what is the value of $\frac{d^2y}{dx^2}$ at the point $(4,3)$?

(A) $-\frac{25}{27}$ (B) $-\frac{7}{27}$ (C) $\frac{7}{27}$ (D) $\frac{3}{4}$ (E) $\frac{25}{27}$

13.

The value of the derivative of $y=\frac{\sqrt[3]{x^2+8}}{\sqrt[4]{2x+1}}$ at $x=0$ is

(A) -1 (B) $-\frac{1}{2}$ (C) 0 (D) $\frac{1}{2}$ (E) 1

14.

If the graph of $y=\frac{ax+b}{x+c}$ has a horizontal asymptote $y=2$ and a vertical asymptote $x=-3$, then $a+c=$

(A) -5 (B) -1 (C) 0 (D) 1 (E) 5

15.

An equation of the line tangent to the graph of $y = \cos(2x)$ at $x = \frac{\pi}{4}$ is

(A) $y - 1 = -\left(x - \frac{\pi}{4}\right)$

(B) $y - 1 = -2\left(x - \frac{\pi}{4}\right)$

(C) $y = 2\left(x - \frac{\pi}{4}\right)$

(D) $y = -\left(x - \frac{\pi}{4}\right)$

(E) $y = -2\left(x - \frac{\pi}{4}\right)$

16.

$\frac{d}{dx}\cos^2(x^3) =$

(A) $6x^2 \sin(x^3)\cos(x^3)$

(B) $6x^2 \cos(x^3)$

(C) $\sin^2(x^3)$

(D) $-6x^2 \sin(x^3)\cos(x^3)$

(E) $-2\sin(x^3)\cos(x^3)$

17.

If $f(x) = (x^2 - 2x - 1)^{\frac{2}{3}}$, then $f'(0)$ is

(A) $\frac{4}{3}$ (B) 0 (C) $-\frac{2}{3}$ (D) $-\frac{4}{3}$ (E) -2

18.

If $f(x) = x\sqrt{2x-3}$, then $f'(x) =$

(A) $\dfrac{3x-3}{\sqrt{2x-3}}$

(B) $\dfrac{x}{\sqrt{2x-3}}$

(C) $\dfrac{1}{\sqrt{2x-3}}$

(D) $\dfrac{-x+3}{\sqrt{2x-3}}$

(E) $\dfrac{5x-6}{2\sqrt{2x-3}}$

19.

If $f(x) = (x-1)^2 \sin x$, then $f'(0) =$

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

20.

If $y = \tan x - \cot x$, then $\dfrac{dy}{dx} =$

(A) $\sec x \csc x$ (B) $\sec x - \csc x$ (C) $\sec x + \csc x$ (D) $\sec^2 x - \csc^2 x$ (E) $\sec^2 x + \csc^2 x$

21.

An equation of the line tangent to the graph of $y = \dfrac{2x+3}{3x-2}$ at the point $(1,5)$ is

(A) $13x - y = 8$ (B) $13x + y = 18$ (C) $x - 13y = 64$

(D) $x + 13y = 66$ (E) $-2x + 3y = 13$

22.

If $x+7y=29$ is an equation of the line normal to the graph of f at the point $(1,4)$, then $f'(1)=$

(A) 7 (B) $\frac{1}{7}$ (C) $-\frac{1}{7}$ (D) $-\frac{7}{29}$ (E) -7

23.

If $x^3+3xy+2y^3=17$, then in terms of x and y, $\frac{dy}{dx}=$

(A) $-\frac{x^2+y}{x+2y^2}$

(B) $-\frac{x^2+y}{x+y^2}$

(C) $-\frac{x^2+y}{x+2y}$

(D) $-\frac{x^2+y}{2y^2}$

(E) $\frac{-x^2}{1+2y^2}$

24.

If $y^2-2xy=16$, then $\frac{dy}{dx}=$

(A) $\frac{x}{y-x}$ (B) $\frac{y}{x-y}$ (C) $\frac{y}{y-x}$ (D) $\frac{y}{2y-x}$ (E) $\frac{2y}{x-y}$

25.

If u, v, and w are nonzero differentiable functions, then the derivative of $\frac{uv}{w}$ is

(A) $\frac{uv'+u'v}{w'}$ (B) $\frac{u'v'w-uvw'}{w^2}$ (C) $\frac{uvw'-uv'w-u'vw}{w^2}$

(D) $\frac{u'vw+uv'w+uvw'}{w^2}$ (E) $\frac{uv'w+u'vw-uvw'}{w^2}$

26.

The $\lim_{h\to 0}\dfrac{\tan 3(x+h)-\tan 3x}{h}$ is

(A) 0 (B) $3\sec^2(3x)$ (C) $\sec^2(3x)$ (D) $3\cot(3x)$ (E) nonexistent

27.

If $y=2\cos\left(\dfrac{x}{2}\right)$, then $\dfrac{d^2y}{dx^2}=$

(A) $-8\cos\left(\dfrac{x}{2}\right)$ (B) $-2\cos\left(\dfrac{x}{2}\right)$ (C) $-\sin\left(\dfrac{x}{2}\right)$ (D) $-\cos\left(\dfrac{x}{2}\right)$ (E) $-\dfrac{1}{2}\cos\left(\dfrac{x}{2}\right)$

28.

If $f(x)=\sin\left(e^{-x}\right)$, then $f'(x)=$

(A) $-\cos(e^{-x})$

(B) $\cos(e^{-x})+e^{-x}$

(C) $\cos(e^{-x})-e^{-x}$

(D) $e^{-x}\cos(e^{-x})$

(E) $-e^{-x}\cos(e^{-x})$

29.

If $f(x)=(x-1)^{\frac{3}{2}}+\dfrac{e^{x-2}}{2}$, then $f'(2)=$

(A) 1 (B) $\dfrac{3}{2}$ (C) 2 (D) $\dfrac{7}{2}$ (E) $\dfrac{3+e}{2}$

30.

$\dfrac{d}{dx}\left(xe^{\ln x^2}\right)=$

(A) $1+2x$ (B) $x+x^2$ (C) $3x^2$ (D) x^3 (E) x^2+x^3

31.

$\lim_{h\to 0} \dfrac{\ln(e+h)-1}{h}$ is

(A) $f'(e)$, where $f(x)=\ln x$

(B) $f'(e)$, where $f(x)=\dfrac{\ln x}{x}$

(C) $f'(1)$, where $f(x)=\ln x$

(D) $f'(1)$, where $f(x)=\ln(x+e)$

(E) $f'(0)$, where $f(x)=\ln x$

32. (Calculator permitted)

Let f be the function given by $f(x)=2e^{4x^2}$. For what value of x is the slope of the line tangent to the graph of f at $(x, f(x))$ equal to 3?

(A) 0.168 (B) 0.276 (C) 0.318 (D) 0.342 (E) 0.551

33.

If $f(x)=\dfrac{e^{2x}}{2x}$, then $f'(x)=$

(A) 1

(B) $\dfrac{e^{2x}(1-2x)}{2x^2}$

(C) e^{2x}

(D) $\dfrac{e^{2x}(2x+1)}{x^2}$

(E) $\dfrac{e^{2x}(2x-1)}{2x^2}$

34.

If $f(x) = \ln\left|x^2 - 1\right|$, then $f'(x) =$

(A) $\left|\dfrac{2x}{x^2-1}\right|$

(B) $\dfrac{2x}{\left|x^2-1\right|}$

(C) $\dfrac{2|x|}{x^2-1}$

(D) $\dfrac{2x}{x^2-1}$

(E) $\dfrac{1}{x^2-1}$

35.

If $y = \arctan(e^{2x})$, then $\dfrac{dy}{dx} =$

(A) $\dfrac{2e^{2x}}{\sqrt{1-e^{4x}}}$ (B) $\dfrac{2e^{2x}}{1+e^{4x}}$ (C) $\dfrac{e^{2x}}{1+e^{4x}}$ (D) $\dfrac{1}{\sqrt{1-e^{4x}}}$ (E) $\dfrac{1}{1+e^{4x}}$

36.

If $e^{f(x)} = 1 + x^2$, then $f'(x) =$

(A) $\dfrac{1}{1+x^2}$ (B) $\dfrac{2x}{1+x^2}$ (C) $2x(1+x^2)$ (D) $2x\left(e^{1+x^2}\right)$ (E) $2x\ln(1+x^2)$

37.

The slope of the line tangent to the graph of $\ln(xy) = x$ at the point where $x = 1$ is

(A) 0 (B) 1 (C) e (D) e^2 (E) $1-e$

38.

If $f(x) = e^{\tan^2 x}$, then $f'(x) =$

(A) $e^{\tan^2 x}$

(B) $\sec^2 x\, e^{\tan^2 x}$

(C) $\tan^2 x\, e^{\tan^2 x - 1}$

(D) $2\tan x \sec^2 x\, e^{\tan^2 x}$

(E) $2\tan x\, e^{\tan^2 x}$

39.

If $f(x) = \ln(e^{2x})$, then $f'(x) =$

(A) 1 (B) 2 (C) $2x$ (D) e^{-2x} (E) $2e^{-2x}$

40.

If $f(x) = e^{3\ln(x^2)}$, then $f'(x) =$

(A) $e^{3\ln(x^2)}$ (B) $\frac{3}{x^2} e^{3\ln(x^2)}$ (C) $6(\ln x) e^{3\ln(x^2)}$ (D) $5x^4$ (E) $6x^5$

41.

$\frac{d}{dx}\left(2^x\right) =$

(A) 2^{x-1} (B) $(2^{x-1})x$ (C) $(2^x)\ln 2$ (D) $(2^{x-1})\ln 2$ (E) $\frac{2x}{\ln 2}$

42.

$\frac{d}{dx} \ln\left|\cos\left(\frac{\pi}{x}\right)\right|$ is

(A) $\frac{-\pi}{x^2 \cos\left(\frac{\pi}{x}\right)}$ (B) $-\tan\left(\frac{\pi}{x}\right)$ (C) $\frac{1}{\cos\left(\frac{\pi}{x}\right)}$

(D) $\frac{\pi}{x}\tan\left(\frac{\pi}{x}\right)$ (E) $\frac{\pi}{x^2}\tan\left(\frac{\pi}{x}\right)$

43.

The slope of the line normal to the graph of $y = 2\ln(\sec x)$ at $x = \frac{\pi}{4}$ is

(A) -2

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) 2

(E) nonexistent

44.

If $f(x) = \left(x^2+1\right)^x$, then $f'(x) =$

(A) $x\left(x^2+1\right)^{x-1}$

(B) $2x^2\left(x^2+1\right)^{x-1}$

(C) $x\ln\left(x^2+1\right)$

(D) $\ln\left(x^2+1\right)+\frac{2x^2}{x^2+1}$

(E) $\left(x^2+1\right)^x\left[\ln\left(x^2+1\right)+\frac{2x^2}{x^2+1}\right]$

45.

If $f(x) = e^x$, then $\ln\left(f'(2)\right) =$

(A) 2 (B) 0 (C) $\frac{1}{e^2}$ (D) $2e$ (E) e^2

46.

If $f(x) = \ln\left(\sqrt{x}\right)$, then $f''(x) =$

(A) $-\frac{2}{x^2}$ (B) $-\frac{1}{2x^2}$ (C) $-\frac{1}{2x}$ (D) $-\frac{1}{2x^{\frac{3}{2}}}$ (E) $\frac{2}{x^2}$

BC Free Response:

1. (AB 2000)

Consider the curve given by $xy^2 - x^3y = 6$.

(a) Show that $\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$.

(b) Find all points on the curve whose x-coordinate is 1, and write an equation for the tangent line at each of these points.

(c) Find the x-coordinate of each point on the curve where the tangent line is vertical.

2. 2008 AB6

Let f be the function given by $f(x)=\dfrac{\ln x}{x}$ for all $x>0$.

(a) Show that the derivative of f is given by $f'(x)=\dfrac{1-\ln x}{x^2}$.

(b) Write an equation for the line tangent to the graph of f at $x=e^2$.

(c) Find the x-coordinate of the point at which $f'(x)=0$.

(d) Find the x-coordinate of the point at which $f''(x)=0$.

(e) Find $\lim\limits_{x\to 0^+} f(x)$.