Name _____ Date _____ Period _____

AP Calculus: Related Rate Worksheet

Calculator permitted. For each of the answers, **Draw and label a diagram**, label all quantities, find the numeric answer, using appropriate units, then give a **'real-world' explanation** of the answer. Show all "position" and "rate" equations.

1. (02 AB) A container has the shape of an open right circular cone. The height of the container is $10cm$ and the diameter of the opening is $10cm$. Water in the container is evaporating so that its depth h is changing at the constant rate of $-\frac{3}{10} cm/hr$.
   a) Find the volume V of water in the container when $h = 5cm$. Indicate units of measure, and
   b) Find the rate of change of the volume of water in the container, with respect to time, when $h = 5cm$. Indicate units of measure.
   c) Show that the rate of change of the volume of water in the container due to evaporation is directly proportional to the exposed surface area of the water. What is the constant of proportionality?

2. The calculus students are in a hot air balloon rising straight up from a level field is tracked by a range finder 500 feet from the lift-off point. At the moment the range finder's elevation angle is $\pi/4$, the angle is increasing at the rate of 0.14 rad/min. How fast are the calculus students in the balloon rising at that moment?

3. The Calculus Cop is in his police cruiser, approaching a right-angled intersection from the North, is chasing a speeding car driven by a calculus student that didn't turn his homework in. The delinquent student has turned the corner and is now moving straight East. When Calculus Cop is 0.6 miles North of the intersection and the backsliding student is 0.8 miles to the East, Calculus Cop determines with his radar that the distance between his awesomeness and the irresponsible student is increasing at 20 mph. If the Calculus Cop is driving at 60 mph at the instant of measurement, what is the speed of the offending student?

4. Water runs into a conical tank at the rate of $9\ ft^3/min$. The tank stands point down and has a height of 10 feet and a base of radius of 5 feet. How fast is the water level rising when the water is 6 feet deep?

5. Mr. Korpi's dad, who is 6 feet tall, walks at the rate of 5 ft/sec toward a streetlight that is 16 feet above the ground. When he is 10 feet from the base of the light,
   a) At what rate is the length of his shadow changing?
   b) At what rate is the tip of his shadow changing?

6. A spherical balloon has a small leak in it. The diameter is decreasing at a constant rate of half an inch per minute. When the diameter is 8 inches: (Assume the balloon is perfectly round.)
   a) How fast is the radius changing?
   b) How fast is the volume changing?
   c) How fast is the surface area changing?
   d) What color is the balloon?

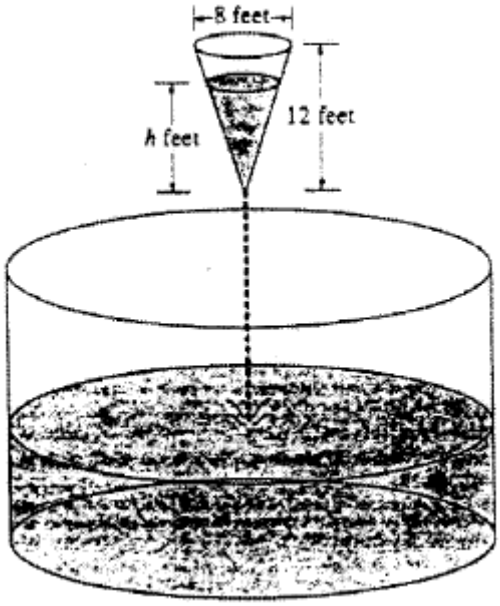
7. A baseball diamond is a 90-foot square. A ball is batted along the third-base line at a constant speed of 100 feet per second. How fast is its distance from first base changing when
   a) it is halfway to third base
   b) it reaches third base?
   c) If the batter heads towards first base a 10mph, did he leave the iron on at home?

8. A point moves along the curve $y = 2x^2 + 1$ in such a way that the y value is decreasing at the rate of 2 units per second. At what rate is x changing when $x = \frac{3}{2}$?

9. In a dangerous calculus experiment, a ladder 10 feet long is leaning against a wall, with the foot of the ladder 8 feet away from the wall. If the foot of the ladder is pulled away from the wall at a constant 3 feet per second:

   b. How fast, in feet per second, is the top of the ladder sliding down the wall?
   c. How fast is the angle the ladder makes with the ground changing? (degrees per second)
   d. How fast is the area formed by the ladder, the ground, and the wall changing?
   e. What is the acceleration of the top of the ladder?
   f. With what speed does the tip of the ladder slam to the ground at the base of the wall?

10. (95 AB) As shown in the figure below, water is draining from a conical tank with height 12 feet and diameter 8 feet into a cylindrical tank that has a base with area 400π square feet. The depth h, in feet, of the water in the conical tank is changing at the rate of $(h\text{-}12)$ feet per minute.



   (a) Write an expression for the volume of water in the conical tank as a function of h.
   (b) At what rate is the volume of water in the conical tank changing when $h = 3$? Indicate units of measure.
   (c) Let y be the depth, in feet, of the water in the cylindrical tank. At what rate is y changing when $h = 3$? Indicate units of measure.

11. A machine is rolling a metal cylinder under pressure. The radius of the cylinder is decreasing at a constant rate of 0.05 inches per second and the volume V is 128π cubic inches. (You can assume that the metal cylinder is a blob of cookie dough, the machinge is a rolling pin, and YOU are the machine operator making cookies for Mr. Korpi). At what rate is the length h changing when the radius r is

   (a) 1.8 inches?
   (b) 2.5 inches?

11. A long level railway bridge passes over a railroad track, which is 100 feet below it and at right angles to it. If a law-abiding calculus student, one who always travels at the posted speed and who always does his homework, is traveling 45 miles per hour is directly above a train going 60 miles per hour below, how fast will they be separating 10 seconds later?