Name____ Date____________________ Per__________

Practice TEST II: AP Calculus: Test—Integration concepts through u-sub

1. If $f'(x) = 12x^2 - 6x + 1$, $f(1) = 5$, then $f(0)$ equals
   (A) 2 (B) 3 C) 4 (D) –1 (E) None of these

2. If $F(x) = \int \left(1 + t^4\right)^{1/3} dt$, then $F'(1)$ equals
   (A) $2^{5/3}$ (B) $2^{2/3}$ (C) $2^{1/3}$ (D) $2^{4/3}$ (E) None of these

3. The volume V of a balloon is changing with respect to time t at a rate given by $\frac{dV}{dt} = 3t^{1/2} + \frac{1}{4}t \quad ft^3/\sec$. If, at $t = 4$, the volume is $20\,ft^3$, then V equals
   (A) $\frac{3}{2}t^{-1/2} + \frac{t}{4} + 19$ (B) $2t^{3/2} + \frac{1}{8}t^2 + 2$ (C) $\frac{9}{2}t^{3/2} + \frac{1}{2}t^2 - 24$ (D) $\frac{9}{4}t^{3/2} + \frac{1}{2}t^2 - 6$ (E) None of these

4. The most general antiderivative of $f(x) = (\cos 3x)(\tan 3x)$ is
   (A) $\frac{1}{3}\tan 3x + C$ (B) $\frac{1}{3}\cot 3x + C$ (C) $-\frac{1}{3}\cos 3x + C$ (D) $\frac{1}{3}\cos 3x + C$ (E) None of these

5. If $f(x) = (\sin x + \cos x)^2$, then an antiderivative of f is
   (A) $\frac{1}{3}(\sin x + \cos x)^3 + C$ (B) $3(\sin x + \cos x) + C$ (C) $\frac{1}{3}\sin^3 x + \frac{1}{3}\cos^3 x + \sin^2 x + C$
   (D) $-\frac{1}{3}(\sin x + \cos x)^3 + C$ (E) $x + \sin^2 x + C$

6. Evaluate the given integral by interpreting it as an area: $\int_0^3 \left(9 - x^2\right)^{1/2} dx$
   (A) 6π (B) 9π (C) $\frac{9\pi}{4}$ (D) 12π (E) None of these

7. If $f(x) = \left(3x^3 + 2\right)$, then find a number c between –1 and 2 such that $\int_{-1}^{2} f(x)dx = 3f(c)$
   (A) $\frac{5}{4}$ (B) $\frac{4}{5}$ (C) $\left(\frac{5}{4}\right)^{1/3}$ (D) $\left(\frac{4}{5}\right)^{1/3}$ (E) None of these

9. $\int_1^2 \left(t^3 - 1\right)^{1/2} t^2 dt$ equals
   (A) $7^{3/2}$ (B) $\frac{2}{9}(7)^{3/2}$ (C) $\frac{1}{2}\left(7^{-1/2}\right)$ (D) $-\frac{2}{9}(7)^{3/2}$ (E) None of these

8. Let f be the function given by $f(x) = x^2 - 2x + 3$. The tangent line to the graph of f at $x = 2$ is used to approximate values of $f(x)$. Which of the following is the greatest value of x for which the error resulting from this tangent line approximation is less than 0.5?
   (A) 2.4 (B) 2.5 (C) 2.6 (D) 2.7 (E) 2.8

9. The average value of $f(x) = x^2 + 3x - 1$ on $[-1, 2]$ equals
   (A) 3 (B) 2 (C) 1.5 (D) 1 (E) None of these

10. The solution to the differential equation $\frac{dy}{dx}=\frac{x^3}{y^2}$, where $y(2)=3$, is

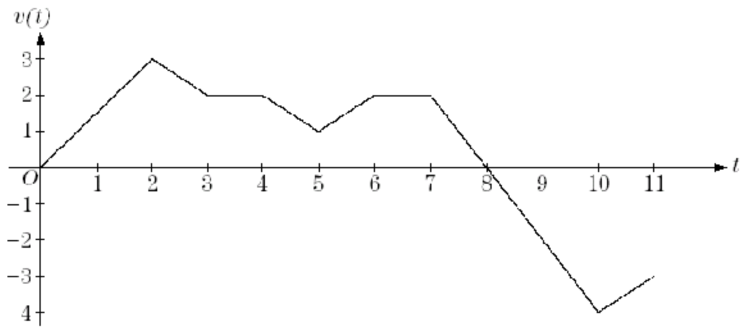
(A) $y=\sqrt[3]{\frac{3}{4}x^4}$ (B) $y=\sqrt[3]{\frac{3}{4}x^4}+\sqrt[3]{15}$ (C) $y=\sqrt[3]{\frac{3}{4}x^4}+15$ (D) $y=\sqrt[3]{\frac{3}{4}x^4+5}$ (E) $y=\sqrt[3]{\frac{3}{4}x^4+15}$

11. $\int x\sqrt{x-1}dx =$

(A) $\frac{3}{2}\sqrt{x-1}-\frac{1}{\sqrt{x-1}}+C$ (B) $\frac{2}{3}(x-1)^{\frac{3}{2}}+\frac{1}{2}(x-1)^{\frac{1}{2}}+C$ (C) $\frac{1}{2}(x-1)^2-(x-1)+C$

(D) $\frac{2}{5}(x-1)^{\frac{5}{2}}-\frac{2}{3}(x-1)^{\frac{3}{2}}+C$ (E) $\frac{1}{2}(x-1)^2+2(x-1)^{\frac{3}{2}}-(x-1)+C$

Use the following graph to answer the next seven questions.



A bug is crawling along a straight wire. The velocity, $v(t)$, in ft/sec of the bug at time t sec, $0 \le t \le 11$, is given in the graph above.

12. According to the graph, at what time t does the bug change direction?

(A) 2 (B) 5 (C) 6 (D) 8 (E) 10

13. According to the graph, at what time t is the speed of the bug greatest?

(A) 2 (B) 5 (C) 6 (D) 8 (E) 10

14. What is the total displacement of the bug has travel on the interval $0 \le t \le 11$?

15. What is the total distance traveled by the bug on the interval $0 \le t \le 11$?

16. What is the bug's average velocity on the interval in the first 4 seconds?

17. What is the bug's average acceleration in the first 4 seconds?

18. What is the bug's acceleration at $t = 1$ seconds

19. If f is continuous for all x, which of the following integrals necessarily have the same value?

I. $\int_a^b f(x)dx$

II. $\int_0^{b-a} f(x+a)dx$

III. $\int_{a+c}^{b+c} f(x+c)dx$

(A) I and II only (B) I and III only (C) II and III only (D) I, II, and III (E) None

20. The table below gives the values for the rate (in gal/sec) at which water flowed into a lake, with readings taken at specific times. A right Riemann sum, with the five subintervals indicated by the data in the table, is used to estimate the total amount of water that flowed into the lake during the time period $0 \le t \le 60$.
   a. What is this estimate?
   b. What is the error between using 5 rectangles vs. 5 **trapezoids**?
   c. What was the average flow rate during the 60 second time period (use the trapezoidal area)

Time (sec)	0	10	25	37	46	60
Rate (gal/sec)	500	400	350	280	200	180

21. Find all possible values of k for which $\int_{-6}^{k} \left(x^3 - 3x\right) dx = 0$

22. $\int_{0}^{5} |3x - 9| dx =$

T or F (if false, explain why or give a counterexample)

23. If $f(t) = \dfrac{\text{calories consumed}}{\text{week}}$ and $dt = \text{week}$, then the definite integral would give total calories consumed over a particular time period.

22. $\int 3x(2x^2 - 1)^4 dx$

23. $\int x^2 \sqrt{x^3 - 1} dx$

24. $\int \dfrac{x^2}{\sqrt{x^3 - 1}} dx$

25. $\int \dfrac{x^2}{\left(x^3 - 1\right)^2} dx$

26. $\int 3\sec^2 2x dx$

27. $\int 5\cot^5 x \csc^2 x dx$

28. $\dfrac{d}{dx} \int_{x^2}^{2x} \dfrac{2t^2 + 3t - 1}{\sqrt{t}} dt$

29. Solve: $\dfrac{dy}{dx} = \dfrac{2x^3}{\sqrt{1 - x^4}}$

30. $\int_{-2}^{6} 2x^2 \sqrt[3]{x + 2} dx$

31. $\int_{1}^{9} \dfrac{1}{\sqrt{x}(1 + \sqrt{x})^2} dx$

32. If $f(x)$ is an even function and $\int_{-17}^{0} f(x) dx = 32.5$, find $2 \int_{-17}^{17} f(x) dx$.

33. I collected some data yesterday and tried to use it to approximate a function $y = f(x)$.

x	**0**	**1/2**	**1**	**3/2**	**2**	**5/2**	**3**
y	**3**	**4**	**1**	**5**	**2**	**3**	**4**

Use my data to approximate $\int_{0}^{3} f(x) dx$ using the following methods:

a. Left end-point Riemann Sums ($n = 6$)
b. Right end-point Riemann Sums ($n = 6$)
c. Midpoint Riemann Sums ($n = 3$)
d. Trapezoidal Rule ($n = 6$)
e. Approximate $f'(1)$ from the table of values.