Name________________________________ Date______________________ Period_______

BC Calculus: Practice TEST: Area, Volumes, Arclengths

Part I: Multiple Choice
NO CALCULATOR ON THIS SECTION

_____ 1.

The base of a solid S is the region enclosed by the graph of $y=\sqrt{\ln x}$, the line $x=e$, and the x-axis. If the cross sections of S perpendicular to the x-axis are squares, then the volume of S is

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) 1 (D) 2 (E) $\frac{1}{3}(e^3-1)$

_____ 2.

When the region enclosed by the graphs of $y=x$ and $y=4x-x^2$ is revolved about the y-axis, the volume of the solid generated is given by

(A) $\pi\int_0^3\left(x^3-3x^2\right)dx$

(B) $\pi\int_0^3\left(x^2-\left(4x-x^2\right)^2\right)dx$

(C) $\pi\int_0^3\left(3x-x^2\right)^2 dx$

(D) $2\pi\int_0^3\left(x^3-3x^2\right)dx$

(E) $2\pi\int_0^3\left(3x^2-x^3\right)dx$

_____ 3.

Let $f(x)=\int_0^{x^2}\sin t\,dt$. At how many points in the closed interval $\left[0,\sqrt{\pi}\right]$ does the instantaneous rate of change of f equal the average rate of change of f on that interval?

(A) Zero
(B) One
(C) Two
(D) Three
(E) Four

_____ 4.

The base of a solid is the region in the first quadrant enclosed by the graph of $y = 2 - x^2$ and the coordinate axes. If every cross section of the solid perpendicular to the y-axis is a square, the volume of the solid is given by

(A) $\pi\int_0^2 (2-y)^2\,dy$

(B) $\int_0^2 (2-y)\,dy$

(C) $\pi\int_0^{\sqrt{2}} \left(2-x^2\right)^2 dx$

(D) $\int_0^{\sqrt{2}} \left(2-x^2\right)^2 dx$

(E) $\int_0^{\sqrt{2}} \left(2-x^2\right) dx$

_____ 5.

$\int_1^e \left(\dfrac{x^2-1}{x}\right) dx =$

(A) $e - \dfrac{1}{e}$ (B) $e^2 - e$ (C) $\dfrac{e^2}{2} - e + \dfrac{1}{2}$ (D) $e^2 - 2$ (E) $\dfrac{e^2}{2} - \dfrac{3}{2}$

_____ 6.

If $\dfrac{dy}{dt} = ky$ and k is a nonzero constant, then y could be

(A) $2e^{kty}$ (B) $2e^{kt}$ (C) $e^{kt} + 3$ (D) $kty + 5$ (E) $\dfrac{1}{2}ky^2 + \dfrac{1}{2}$

_____ 7.

What is the average value of $y = x^2\sqrt{x^3+1}$ on the interval $[0, 2]$?

(A) $\dfrac{26}{9}$ (B) $\dfrac{52}{9}$ (C) $\dfrac{26}{3}$ (D) $\dfrac{52}{3}$ (E) 24

_____ 8.

If $\dfrac{dy}{dx} = \sin x \cos^2 x$ and if $y = 0$ when $x = \dfrac{\pi}{2}$, what is the value of y when $x = 0$?

(A) -1 (B) $-\dfrac{1}{3}$ (C) 0 (D) $\dfrac{1}{3}$ (E) 1

_____ 9.

Which of the following integrals gives the length of the graph of $y = \tan x$ between $x = a$ and $x = b$, where $0 < a < b < \frac{\pi}{2}$?

(A) $\int_a^b \sqrt{x^2 + \tan^2 x}\, dx$

(B) $\int_a^b \sqrt{x + \tan x}\, dx$

(C) $\int_a^b \sqrt{1 + \sec^2 x}\, dx$

(D) $\int_a^b \sqrt{1 + \tan^2 x}\, dx$

(E) $\int_a^b \sqrt{1 + \sec^4 x}\, dx$

_____ 10.

A region in the plane is bounded by the graph of $y = \frac{1}{x}$, the x-axis, the line $x = m$, and the line $x = 2m$, $m > 0$. The area of this region

(A) is independent of m.

(B) increases as m increases.

(C) decreases as m increases.

(D) decreases as m increases when $m < \frac{1}{2}$; increases as m increases when $m > \frac{1}{2}$.

(E) increases as m increases when $m < \frac{1}{2}$; decreases as m increases when $m > \frac{1}{2}$.

_____ 11.

The region in the first quadrant bounded by the graph of $y = \sec x$, $x = \frac{\pi}{4}$, and the axes is rotated about the x-axis. What is the volume of the solid generated?

(A) $\frac{\pi^2}{4}$ (B) $\pi - 1$ (C) π (D) 2π (E) $\frac{8\pi}{3}$

_____ 12.

The region R in the first quadrant is enclosed by the lines $x = 0$ and $y = 5$ and the graph of $y = x^2 + 1$. The volume of the solid generated when R is revolved about the y-axis is

(A) 6π (B) 8π (C) $\frac{34\pi}{3}$ (D) 16π (E) $\frac{544\pi}{15}$

_____ 13.

The length of the curve $y = x^3$ from $x = 0$ to $x = 2$ is given by

(A) $\int_0^2 \sqrt{1+x^6}\,dx$ (B) $\int_0^2 \sqrt{1+3x^2}\,dx$ (C) $\pi\int_0^2 \sqrt{1+9x^4}\,dx$

(D) $2\pi\int_0^2 \sqrt{1+9x^4}\,dx$ (E) $\int_0^2 \sqrt{1+9x^4}\,dx$

_____ 14.

The area of the region enclosed by the graphs of $y = x^2$ and $y = x$ is

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{5}{6}$ (E) 1

II. Free Response
(CALCULATOR PERMITTED)
1. (2007-BC1)

Let R be the region in the first and second quadrants bounded above by the graph of $y = \dfrac{20}{1+x^2}$ and below by the horizontal line $y = 2$.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is rotated about the x-axis.

(c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are semicircles. Find the volume of this solid.

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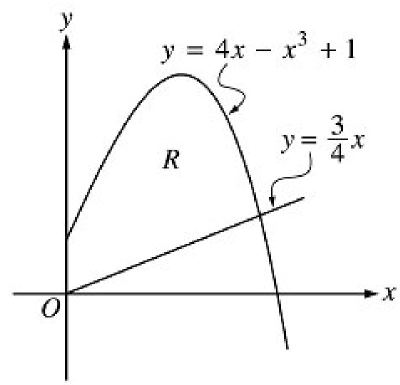
2. (2002B-BC3)

Let R be the region in the first quadrant bounded by the y-axis and the graphs of $y = 4x - x^3 + 1$ and $y = \dfrac{3}{4}x$.



(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the x-axis.

(c) Write an expression involving one or more integrals that gives the perimeter of R. Do not evaluate.

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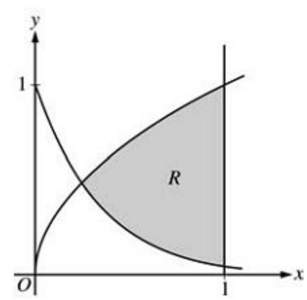
3. (2003-BC1)

Let R be the shaded region bounded by the graphs of $y = \sqrt{x}$ and $y = e^{-3x}$ and the vertical line $x = 1$, as shown in the figure above.



(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 1$.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a rectangle whose height is 5 times the length of its base in region R. Find the volume of this solid.

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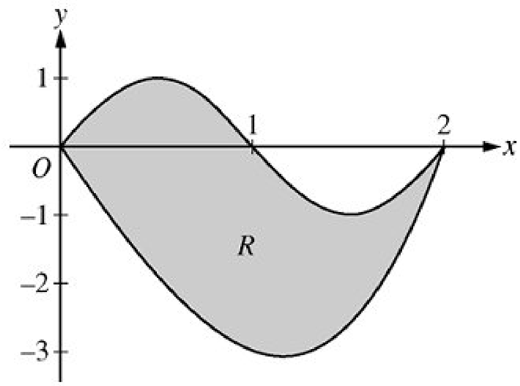
4. (2008-BC1)



Let R be the region bounded by the graphs of $y = \sin(\pi x)$ and $y = x^3 - 4x$, as shown in the figure above.

(a) Find the area of R.

(b) The horizontal line $y = -2$ splits the region R into two parts. Write, but do not evaluate, an integral expression for the area of the part of R that is below this horizontal line.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

(d) The region R models the surface of a small pond. At all points in R at a distance x from the y-axis, the depth of the water is given by $h(x) = 3 - x$. Find the volume of water in the pond.

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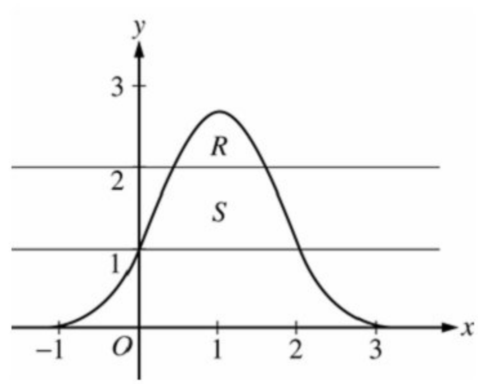
5. (2007B-BC1)

Let R be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal line $y = 2$, and let S be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal lines $y = 1$ and $y = 2$, as shown above.

(a) Find the area of R.

(b) Find the area of S.

(c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 1$.



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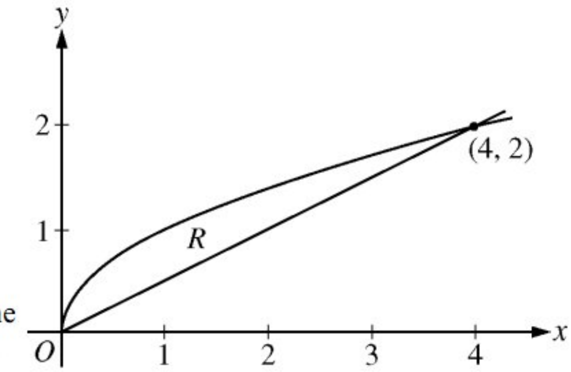
6. (2009B-AB4)

Let R be the region bounded by the graphs of $y = \sqrt{x}$ and $y = \frac{x}{2}$, as shown in the figure above.



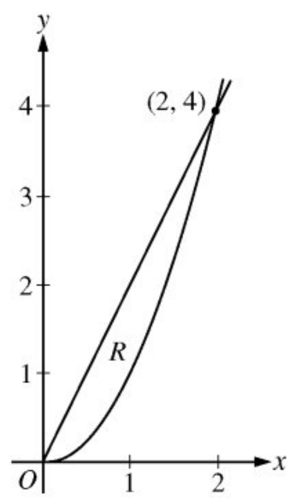
(a) Find the area of R.

(b) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are squares. Find the volume of this solid.

(c) Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y = 2$.

7. (2009-AB4)

Let R be the region in the first quadrant enclosed by the graphs of $y = 2x$ and $y = x^2$, as shown in the figure above.



(a) Find the area of R.

(b) The region R is the base of a solid. For this solid, at each x the cross section perpendicular to the x-axis has area $A(x) = \sin\left(\frac{\pi}{2}x\right)$. Find the volume of the solid.

(c) Another solid has the same base R. For this solid, the cross sections perpendicular to the y-axis are squares. Write, but do not evaluate, an integral expression for the volume of the solid.

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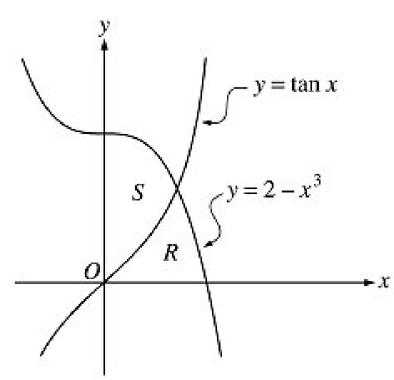
8. (2001-AB1)

Let R and S be the regions in the first quadrant shown in the figure above. The region R is bounded by the x-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$. The region S is bounded by the y-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$.



(a) Find the area of R.

(b) Find the area of S.

(c) Find the volume of the solid generated when S is revolved about the x-axis.
