SERIES DAY 3

Geometric Series, nth Term Test for Divergence, and Telescoping Series

Geometric Series Test

A geometric series is in the form $\sum_{n=0}^{\infty} a_1 r^n$ or $\sum_{n=1}^{\infty} a_1 r^{n-1}, a \neq 0$

The geometric series **diverge**s if $|r| \geq 1$.

If $|r| < 1$, the series **converge**s to the sum $S = \frac{a_1}{1-r}$.

NOTE: this formula works only when the first term is a_1, otherwise you can adapt it by subtracting out "missing" terms!!

Ex. Determine whether the following series converge or diverge.

(a) $\sum_{n=1}^{\infty} \frac{3}{2^n}$

(b) $\sum_{n=1}^{\infty} \left(\frac{3}{2}\right)^n$

***n*th Term Test for Divergence**

If $\lim_{n\to\infty} a_n \neq 0$, then the series $\sum_{n=1}^{\infty} a_n$ diverges.

(think about it, it should make perfect sense!)

Note: This does NOT say that if $\lim_{n\to\infty} a_n = 0$, then the series DOES converge. This test can only be used to prove that a series diverges (hence the name.) If $\lim_{n\to\infty} a_n = 0$, then this test doesn't tell us anything, is inconclusive, doesn't work, fails, etc. . . . We MUST use another test. This test is a GREAT time-saver. Always perform it **FIRST**, not second, but FIRST!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

Ex. Determine whether the following series converge or diverge.

(a) $\sum_{n=1}^{\infty} \frac{2n+3}{3n-5}$

(b) $\sum_{n=1}^{\infty} \frac{n!}{2n!+1}$

(c) $\sum_{n=1}^{\infty} \frac{3^n-2}{3^n}$

A series such as $\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\cdots$ is called a **telescoping series** because it collapses to one term or a few terms. If a series collapses to a finite sum, then it converges by the **Telescoping Series Test**.

Ex. Determine whether the following series converges or diverges.

(a) $\sum_{n=1}^{\infty}\left(\frac{1}{2n+1}-\frac{1}{2n+3}\right)$

(b) $\sum_{n=1}^{\infty}\frac{1}{n(n+1)}$

<u>Integral Test and *p*-Series</u>

Integral Test

If f is **D**ecreasing, **C**ontinuous, and **P**ositive (**D**ogs **C**uss in **P**rison!) for $x \geq 1$ AND $a_n = f(x)$, then $\sum_{n=1}^{\infty} a_n$ and $\int_1^{\infty} f(x)dx$ either BOTH converge or diverge.

Note: This does NOT mean that the series converges the value of the definite integral!!!!!!

If the series converges to S, then the remainder, $R_N = |S - S_N|$ is bounded by $0 \leq R_N \leq \int_N^{\infty} f(x)dx$.

Ex. Determine whether the following series converge or diverge.

(a) $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$

(b) $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$

Ex. Approximate the sum of the convergent series $\sum_{n=1}^{\infty} \frac{1}{n^4}$ by using six terms. Include an estimate of the maximum error for your approximation.

p-series

A series of the form $\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \cdots + \frac{1}{n^p} + \cdots$ is called a p-series, where p is a positive constant.

For $p = 1$, the series $\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} + \cdots$ is called the **harmonic series**.

What values of p would cause $\sum_{n=1}^{\infty} \frac{1}{n^p}$ to converge? to diverge?

Let $p = 1$:

Let $p = 1.1$:

Let $p = 0.9$:

p-Series Test

The p-series $\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \cdots + \frac{1}{n^p} + \cdots$

a)

b)

c)

Ex. $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}}$