SERIES DAY 5

Ratio and Root Tests

Ratio Test

Let $\sum_{n=1}^{\infty} a_n$ be a series of nonzero terms.

1. $\sum_{n=1}^{\infty} a_n$ converges if $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| < 1$
2. $\sum_{n=1}^{\infty} a_n$ diverges if $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| > 1$
3. The ratio test is inconclusive if $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 1$

Series involving expressions that grow very rapidly such as factorials and/or exponential work especially well with the Ratio Test.

Ex. $\sum_{n=1}^{\infty} \frac{2^n}{n!}$

Ex. $\sum_{n=1}^{\infty} \frac{n^2 3^{n+1}}{2^n}$

Ex. $\sum_{n=1}^{\infty} \frac{(n+1)!}{3^n}$

Root Test

1. $\sum_{n=1}^{\infty} a_n$ converges if $\lim_{n\to\infty} \sqrt[n]{|a_n|} < 1$
2. $\sum_{n=1}^{\infty} a_n$ diverges if $\lim_{n\to\infty} \sqrt[n]{|a_n|} > 1$
3. The Root Test is inconclusive if $\lim_{n\to\infty} \sqrt[n]{|a_n|} = 1$

If the entire rule of sequence can be written as power of *n*, the Root Test is hard to beat!

Ex. $\sum_{n=1}^{\infty} \frac{e^{2n}}{n^n}$

Ex. $\sum_{n=1}^{\infty} \left(\frac{3n+4}{2n}\right)^n$