SERIES DAY 6

Taylor Polynomials and Approximations

Polynomial functions can be used to approximate other elementary functions such as $\sin x$, e^x , and $\ln x$. On your calculator graph:

$$y1 = \sin x$$

$$y2: x - \frac{x^3}{3!} + \frac{x^5}{5!}$$

And compare.

Definition of an *n*th-degree Taylor polynomial:

If f has n derivatives at $x = c$, then the polynomial

$$P_n(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \cdots + \frac{f^{(n)}(c)}{n!}(x-c)^n$$

is called the *n*th-degree Taylor polynomial for f at c, named after Brook Taylor, an English mathematician.

Note: A first-degree Taylor polynomial is a tangent line to f at c.

If $c = 0$, then $P_n(x) = f(0) + f'(0)(x) + \frac{f''(0)}{2!}x^2 + \cdots + \frac{f^{(n)}(0)}{n!}x^n$ is called the *n*th-degree Maclaurin polynomial for f , named after Scottish mathematician, Colin Maclaurin.

Note: Maclaurin not only got his name on a very specific case of Taylor's work, but he also had big hair.

Ex. Find the Maclaurin polynomial of degree $n = 6$ for $f(x) = \cos x$. Then use $P_6(x)$ to approximate the value of $\cos(0.1)$.

Ex. Find the Taylor polynomial of degree $n = 6$ for $f(x) = \ln x$ at $c = 1$. Then use $P_6(x)$ to approximate the value of $\ln(1.1)$

Ex. Suppose that g is a function which has continuous derivatives, and that $g(2)=3$, $g'(2)=-4$, $g''(2)=7$, $g'''(2)=-5$. Write the Taylor polynomial of degree 3 for g centered at 2.

Ex. Use the Taylor approximation of $e^x \approx 1+x+\frac{x^2}{2!}+\frac{x^3}{3!}$ for x near 0 to find $\lim_{x\to 0}\frac{e^x-1}{2x}$.

Ex. Given that $P_2(x)=a+bx+c^2$ is the second-degree Taylor polynomial for f about $x=0$, what can you say about the signs of a, b, and c if f has the graph pictured at the right? Justify your answer.