SERIES DAY 7
POWER SERIES

If x is a variable, then an infinite series of the form $\sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + \cdots a_n x^n + \cdots$ is called a power series.

$\sum_{n=0}^{\infty} a_n (x-c)^n = a_0 + a_1 (x-c) + a_2 (x-c)^2 + \cdots a_n (x-c)^n + \cdots$ is a power series centered at c, where c is a constant.

For a power series centered at c, precisely one of the following is true:

1) The series converges only at c (ALL power series converge at their center!!)
2) THE series converges for all x.
3) There exists an $R > 0$ such that the series converges for $|x-c| < R$ and diverges for $|x-c| > R$.

R is called the radius of convergence of the power series.
In part 1) the radius is 0.
In part 2), the radius is ∞
The corresponding domain, $[(c-R, c+R)]$, is called the interval of convergence.
Note: to determine if the endpoints are included or not, we must test each endpoint independently.
Note2: We typically use the RATIO TEST to determine the radius of convergence.

Ex. Find the radius of convergence and the interval of convergence. Be sure to check the endpoints.

(a) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} (x-5)^n}{n2^n}$

(b) $\sum_{n=0}^{\infty}\left(\frac{x}{3}\right)^{n}$

(c) $\sum_{n=0}^{\infty}\frac{(-1)^{n}x^{2n+1}}{(2n+1)!}$

(d) $\sum_{n=0}^{\infty}n!(x-3)^{n}$