SERIES DAY 8
TAYLOR SERIES

Taylor Series centered at $x = c$:

$$f(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \cdots + \frac{f^{(n)}(c)}{n!}(x-c)^n + \cdots = \sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!}(x-c)^n$$

If $c = 0$, the series is called a Maclaurin series.
Ex. Find a Taylor series for $f(x) = e^{5x}$ centered at $c = 2$. Give the first four nonzero terms and the general term.

There are three special Maclaurin series you must know. These are the series for e^x, $\sin x$, and $\cos x$.

To derive a series for e^x:

To derive a series for $\sin x$:

To derive a series for $\cos x$:

You can manipulate these three special series (or any series we are given) to find other series by using the following techniques:

1) Substitute into a series
2) Multiply or divide the series by a constant and/or a variable
3) Add or subtract two series
4) Differentiate or integrate a series
5) Recognize the series as the sum of a geometric power series

Ex. Find a Maclaurin series for $f(x) = \sin\left(x^2\right)$. Find the first four nonzero terms and the general term.

Ex. Find a Maclaurin series for $g(x) = x\cos x$. Find the first four nonzero terms and the general term.

Ex. Find a Maclaurin series for $h(x) = \dfrac{e^x + e^{-x}}{2}$. Find the first four nonzero terms and the general term.