

## POLYNOMIAL APPROXIMATIONS AND SERIES

From the AP Calculus Course Description:

Concept of series. A series is defined as a sequence of partial sums, and convergence is defined in terms of the limit of the sequence of the partial sums. Technology can be used to explore convergence or divergence.

Series of constants.

- Motivating examples, including decimal expansion
- Geometric series with applications
- The harmonic series
- Alternating series with error bound
- Terms of series as areas of rectangles and the relationship to improper integrals, including the integral test and its use in testing for convergence of  $p$ -series
- The ratio test for convergence and divergence
- Comparing series to test for convergence or divergence (direct & limit comparisons)

Taylor series.

- Taylor polynomial approximation with graphical demonstration of convergence. (For example, viewing graphs of various Taylor polynomials of the sine function approximating the sine curve.)
- Maclaurin series and the general Taylor series centered at  $x = a$
- Maclaurin series for the functions  $e^x$ ,  $\sin x$ ,  $\cos x$ ,  $\ln(x+1)$ , and  $\frac{1}{1-x}$
- Formal manipulation of Taylor series and shortcuts to computing Taylor series, including substitution, differentiation, antidifferentiation, and the formation of new series from known series
- Functions defined by power series
- Radius and interval of convergence of power series
- Lagrange error bound for Taylor polynomials

## SEQUENCES ON THE GRAPHING CALCULATOR

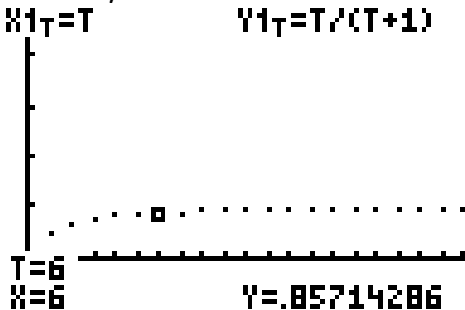
Sequences can be graphed on any graphing calculator by using parametric mode with a T-step of 1 with the graphing mode set to "dot" instead of "connected" or by using sequence mode.

Ex.  $a_n = \left\{ \frac{n}{n+1} \right\}$

**Parametric Mode on TI-83, TI-89, etc.:**

<pre>Plot1 Plot2 Plot3 X1T=T Y1T=T/(T+1) X2T= Y2T= X3T= Y3T= X4T=</pre>	<pre>WINDOW Tmin= Tmax=20 Tstep=1 Xmin=-1 Xmax=20 Xscl=1 Ymin=-1</pre>	<pre>WINDOW Tstep=1 Xmin=-1 Xmax=20 Xscl=1 Ymin=-1 Ymax=5 Yscl=1</pre>
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Press "Graph" then "Trace" to see the terms of the sequence.  
Be sure you're in "dot" mode so the dots won't be connected.



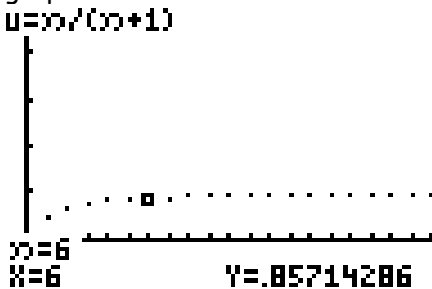
**Sequence Mode on the TI-83:**

<pre>Plot1 Plot2 Plot3 nMin=1 u(n)=n/(n+1) u(nMin)=1/2 v(n)= v(nMin)= w(n)= w(nMin)=</pre>	<pre>WINDOW nMin=1 nMax=20 PlotStart=1 PlotStep=1 Xmin=-1 Xmax=20 Xscl=1</pre>	<pre>WINDOW PlotStep=1 Xmin=-1 Xmax=20 Xscl=1 Ymin=-1 Ymax=5 Yscl=</pre>
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NOTE: press "2<sup>nd</sup>" "7" to get the "u" and the "x,t,θ,n" key to get the "n."

Press "graph" and then "trace" to see the terms of the sequence.

Depending on the operating system of your TI-83, you may need to set it to "dot" mode so that the dots on the graph are not connected.



CALCULUS BC

LAB ON INVESTIGATING SEQUENCES

Let  $\{a_n\}$  be a sequence of real numbers.

Possibilities:

- 1) If  $\lim_{n \rightarrow \infty} a_n = \infty$ , then  $\{a_n\}$  diverges to infinity
- 2) If  $\lim_{n \rightarrow \infty} a_n = -\infty$ , then  $\{a_n\}$  diverges to negative infinity
- 3) If  $\lim_{n \rightarrow \infty} a_n = c$ , an finite real number, then  $\{a_n\}$  converges to  $c$
- 4) If  $\lim_{n \rightarrow \infty} a_n$  oscillates between two fixed numbers, then  $\{a_n\}$  diverges by oscillation

Use your graphing calculator to determine whether the following sequences converge or diverge. Look at both the graphs of the sequence and the table of  $y$ -values as  $n$  increases ( $n = 1, 2, 3, \dots$ )

1.  $a_n = \frac{n}{n+1}$

6.  $a_n = \frac{n!}{(n+2)!}$

2.  $a_n = \frac{1}{2^n}$

7.  $a_n = \frac{n!}{(n+1)!}$

3.  $a_n = 3 + (-1)^n$

8.  $a_n = \frac{n + (-1)^n}{n}$

4.  $a_n = \frac{n}{1-2n}$

9.  $a_n = \frac{(-1)^n(n-1)}{n}$

5.  $a_n = \frac{\ln n}{n}$

10.  $a_n = \left(1 + \frac{1}{n}\right)^n$

A **sequence** is a function whose domain is the set of positive integers:

$$a_1, a_2, a_3, \dots, a_n, \dots$$

$a_n$  is called the  **$n$ th term** (or rule of sequence), and we denote the sequence by  $\{a_n\}$ .

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Ex. Write the first five terms of the sequence

(a)  $a_n = 3^n$

(b)  $a_n = \frac{2}{n!}$

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Ex. Write an expression for the  $n$ th term.

(a) 3, 8, 13, 18, ...

(b) 5, -15, 45, -135, ...

(c) 1, 4, 9, 16, 25, ...

(d) 4, 10, 28, 82, ...

(e)  $\frac{2}{1}, \frac{3}{3}, \frac{4}{5}, \frac{5}{7}, \frac{6}{9}, \dots$

If a sequence has a finite limit as  $n \rightarrow \infty$ , we say that the sequence **converges**. If a sequence does not have a finite limit as  $n \rightarrow \infty$ , then it **diverges**.

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Ex. Determine whether the following sequences converge or diverge. If they converge, state the value to which they converge.

(a)  $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \dots, \frac{n}{n+1}, \dots$

(b)  $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots, \frac{1}{2^n}, \dots$

(c)  $a_n = 3 + (-1)^n$

(d)  $a_n = \frac{n}{1-2n}$

(e)  $a_n = \frac{\ln n}{n}$

(f)  $a_n = \frac{n!}{(n+2)!}$

(g)  $a_n = \frac{n!}{(n-1)!}$