CALCULUS BC
WORKSHEET ON SERIES

Work the following on notebook paper. Use your calculator only on 10(b).

1. Which of the following is a term in the Taylor series about $x=0$ for the function $f(x)=\cos(2x)$?

(A) $-\frac{1}{2}x^2$ (B) $-\frac{4}{3}x^3$ (C) $\frac{2}{3}x^4$ (D) $\frac{1}{60}x^5$ (E) $\frac{4}{45}x^6$

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2. Find the values of $x$ for which the series $\sum_{n=1}^{\infty}\frac{(x-2)^n}{n(-3)^n}$ converges.

(A) $x=2$ (B) $-1\le x<5$ (C) $-1<x\le 5$ (D) $-1<x<5$ (E) All real numbers

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3. Let $f(x)=\sum_{n=1}^{\infty}(\cos x)^{3n}$. Evaluate $f\left(\frac{2\pi}{3}\right)$.

(A) $-\frac{1}{7}$ (B) $-\frac{1}{9}$ (C) $\frac{1}{7}$ (D) $\frac{8}{9}$ (E) The series diverges.

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4. Find the sum of the geometric series $\frac{9}{8}-\frac{3}{4}+\frac{1}{2}-\frac{1}{3}+...$

(A) $\frac{3}{5}$ (B) $\frac{5}{8}$ (C) $\frac{13}{24}$ (D) $\frac{27}{8}$ (E) $\frac{27}{40}$

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5. The series $x+x^3+\frac{x^5}{2!}+\frac{x^7}{3!}+...+\frac{x^{2n+1}}{n!}+...$ is the Maclaurin series for

(A) $x\ln(1+x^2)$ (B) $x\ln(1-x^2)$ (C) $e^{x^2}$ (D) $xe^{x^2}$ (E) $x^2e^{x^2}$

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6. The coefficient of $x^3$ in the Taylor series for $e^{2x}$ at $x=0$ is

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{4}{3}$ (E) $\frac{8}{3}$

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7. The Taylor polynomial of order 3 at $x=0$ for $f(x)=\sqrt{1+x}$ is

(A) $1+\frac{x}{2}-\frac{x^2}{4}+\frac{3x^3}{8}$ (B) $1+\frac{x}{2}-\frac{x^2}{8}+\frac{x^3}{16}$ (C) $1-\frac{x}{2}+\frac{x^2}{8}-\frac{x^3}{16}$

(D) $1+\frac{x}{2}-\frac{x^2}{8}+\frac{x^3}{8}$ (E) $1-\frac{x}{2}+\frac{x^2}{4}-\frac{3x^3}{8}$

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8. The function $f$ has a Taylor series about $x = 2$ that converges to $f(x)$ for all $x$ in the interval of convergence. The nth derivative of $f$ at $x = 2$ is given by $f^{(n)}(2) = \dfrac{(n+1)!}{3^n}$ for $n \geq 1$, and $f(2) = 1$.

(a) Write the first four terms and the general term of the Taylor series for $f$ about $x = 2$.

(b) Find the radius of convergence for the Taylor series for $f$ about $x = 2$. Show the work that leads to your answer.

(c) Let $g$ be a function satisfying $g(2) = 3$ and $g'(x) = f(x)$ for all $x$. Write the first four terms and the general term of the Taylor series for $g$ about $x = 2$.

(d) Does the Taylor series for $g$ as defined in part (c) converge at $x = -2$? Give a reason for your answer.

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9. The function $f$ is defined by the power series

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n+1)!} = 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \ldots + \frac{(-1)^n x^{2n}}{(2n+1)!} + \ldots$$

for all real numbers $x$.

(a) Find $f'(0)$ and $f''(0)$. Determine whether $f$ has a local maximum, a local minimum, or neither at $x = 0$. Give a reason for your answer.

(b) Show that the second-degree Taylor polynomial for $f$ about $x = 0$ approximates $f(1)$ with error less than $\dfrac{1}{100}$.

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10. Let $f$ be a function that has derivatives of all orders on the interval $(-1, 1)$. Assume that $f(0) = 6$, $f'(0) = 8$, $f''(0) = 30$, $f'''(0) = 48$, and $\left|f^{(n)}(x)\right| \leq 75$ for all $x$ in (0, 1).

(a) Write a third-degree Taylor polynomial for $f$ about $x = 0$.

(b) Use your answer to (a) to estimate the value of $f(0.2)$. What is the maximum possible error in making this estimate? Justify your answer.

11. Let $f$ be the function given by $f(x) = \cos\left(3x + \dfrac{3\pi}{4}\right)$, and let $P(x)$ be the third-degree Taylor polynomial for $f$ about $x = 0$.

(a) Find $P(x)$.

(b) Use the Lagrange error bound to show that $\left|f\left(\dfrac{1}{6}\right) - P\left(\dfrac{1}{6}\right)\right| \leq \dfrac{1}{300}$.

(c) Let $G$ be the function given by $G(x) = \int_0^x f(t)\,dt$. Write the third-degree Taylor polynomial for $G$ about $x = 0$.