CALCULUS BC
REVIEW SHEET ON SERIES

Work the following on notebook paper. Use your calculator only on 4(c), 10, 15, and 16.

Find the radius and interval of convergence.

1. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n)!}$

2. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} (x-3)^n}{n4^n}$

3. (a) Find the interval of convergence for $f(x)=\sum_{n=0}^{\infty} \frac{x^{n+1}}{(n+1)^2}$.

   (b) Write the first four nonzero terms and the general term for $f'(x)$, and find its interval of convergence.

4. (a) Find a power series for $f(x)=\frac{1}{1+x^2}$ centered at $x=0$. Write the first four nonzero terms and the general term.

   (b) Use your answer to (a) to find the first four nonzero terms and the general term for $g(x)=\arctan x$.

   (c) Use your answer to (b) to approximate $\arctan\frac{1}{3}$, using $R_N \le 0.001$.

For problems 5 – 8, write the first four nonzero terms and the general term.

5. Maclaurin series for $f(x)=\sin\left(x^3\right)$

6. Power series for $g(x)=\frac{x}{1+2x}$ centered at $x=0$

7. Taylor series for $h(x)=x\cos x$ centered at $x=0$

8. Taylor series for $f(x)=\ln(3-x)$ centered at $x=2$

9. Suppose $f(x)$ is approximated near $x=0$ by a fifth-degree Taylor polynomial $P_5(x)=2x-5x^3+4x^5$. Give the value of:

   (a) $f''(0)$ (b) $f'''(0)$ (c) $f^{(5)}(0)$

10. Suppose g is a function which has continuous derivatives and that $g(4)=2,\ g'(4)=-3,\ g''(4)=5.$ Write a Taylor polynomial of degree 2 for g, centered at $x=4$, and use it to approximate $g(4.1)$.

11. Suppose $P_4(x)=a+bx+cx^2$ is the second-degree Taylor polynomial for the function f about $x=0$. What can you say about the signs of a, b, and c if f has the graph pictured on the right?

TURN->>>

By recognizing the following as a Taylor series evaluated at a particular value of x, find the sum of each of the following convergent series.

12. $1+\frac{3}{1!}+\frac{9}{2!}+\frac{27}{3!}+...+\frac{3^n}{n!}+...$

13. $1+\frac{1}{5}+\left(\frac{1}{5}\right)^2+\left(\frac{1}{5}\right)^3+...+\left(\frac{1}{5}\right)^n+...$

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14. Use power series to evaluate $\lim_{x\to 0}\frac{e^x-e^{-x}}{x}$.

15. Use a Taylor polynomial of degree 5 for $\sin x$ about $x=0$ to estimate $\int_0^1 \frac{\sin x}{x}\,dx$.

16. The function f has derivatives of all orders for all real numbers x. Assume $f(3)=-5,\ f'(3)=2,\ f''(3)=-7,\ f'''(3)=9$.

(a) Write the third-degree Taylor polynomial for f about $x=3$, and use it to approximate $f(2.6)$.

(b) The fourth derivative of f satisfies the inequality $\left|f^{(4)}(x)\right|\le 5$ for all x in the closed interval [2.6, 3]. Use the Lagrange error bound on the approximation to $f(2.6)$ found in part (a) to explain whether or not $f(2.6)$ can equal -6.

(c) Write the fourth-degree Taylor polynomial, $Q(x)$, for $g(x)=f\left(x^2+3\right)$ about $x=0$.

(d) Use your answer to (c) to determine whether g has a relative maximum, a relative minimum, or neither at $x=0$. Justify your answer.

17. The Taylor series about $x=4$ for a certain function f converges to $f(x)$ for all x in the interval of convergence. The nth derivative of f at $x=4$ is given by $f^{(n)}(4)=\frac{(-1)^n n!}{3^n(n+1)}$ for $n\ge 1$ and $f(4)=2$.

(a) Write the third-degree Taylor polynomial for f about $x=4$.

(b) Find the radius of convergence.

(c) Use the series found in (a) to approximate $f(5)$ with an error less than 0.02.