WORKSHEET 6: TAYLOR POLYNOMIALS

Work the following on notebook paper. Use your calculator only on problems 9, 10 and 12. Show all work.

On problems 1-5, find a Maclaurin polynomial of degree *n* for each of the following.

1. $f(x) = e^{-x}$, $n = 3$
2. $f(x) = e^{2x}$, $n = 4$
3. $f(x) = \sin x$, $n = 7$
4. $f(x) = xe^{2x}$, $n = 4$
5. $f(x) = \dfrac{1}{x+1}$, $n = 5$

On problems 6-8, find a Taylor polynomial of degree *n* centered at $x = c$ for each of the following.

6. $f(x) = \dfrac{1}{x}$, $n = 5$, $c = 1$
7. $f(x) = \ln x$, $n = 5$, $c = 1$
8. $f(x) = \cos x$, $n = 6$, $c = \dfrac{\pi}{4}$

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9. Use your answer from problem 1 to approximate $f\left(\dfrac{1}{2}\right)$ to four decimal places.
10. Use your answer from problem 7 to approximate $f(1.2)$ to four decimal places.
11. Suppose that function $f(x)$ is approximated near $x = 0$ by a sixth-degree Taylor polynomial $P_6(x) = 3x - 4x^3 + 5x^6$. Give the value of:

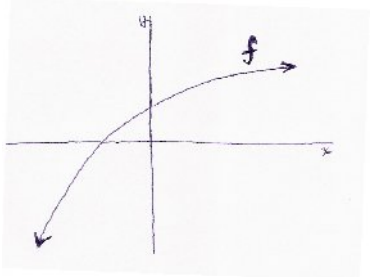
    (a) $f(0)$ (b) $f'(0)$ (c) $f'''(0)$ (d) $f^{(5)}(0)$ (e) $f^{(6)}(0)$
12. Suppose that *g* is a function which has continuous derivatives, and that $g(5) = 3$, $g'(5) = -2$, $g''(5) = 1$, $g'''(5) = -3$

    (a) What is the Taylor polynomial of degree 2 for *g* near 5? What is the Taylor polynomial of degree 3 near 5?

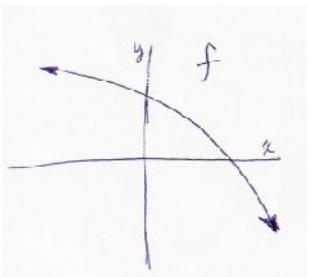
    (b) Use the two polynomials that you found in part (a) to approximate $g(4.9)$.

For problems 13-16, suppose that $P_2(x) = a + bx + cx^2$ is the second degree Taylor polynomial for the function *f* about $x = 0$. What can you say about the signs of *a*, *b*, and *c*, if *f* has the graphs given below?

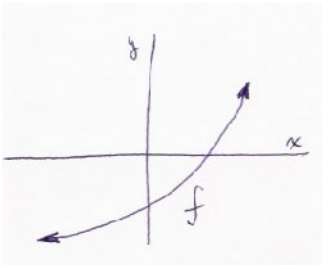
13.



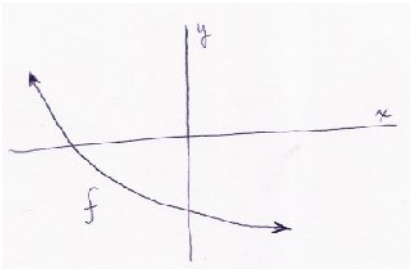
14.



15.



16.



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17. Show how you can use the Taylor approximation $\sin x \approx x - \dfrac{x^3}{3!}$ for *x* near 0 to find $\lim\limits_{x \to 0} \dfrac{\sin x}{x}$.
18. Use the fourth-degree Taylor approximation of $\cos x \approx 1 - \dfrac{x^2}{2!} + \dfrac{x^4}{4!}$ for *x* near 0 to find $\lim\limits_{x \to 0} \dfrac{1 - \cos x}{x}$.
19. Estimate the integral $\int_0^1 \dfrac{\sin t}{t} dt$ using a Taylor polynomial for $\sin t$ about $t = 0$ of degree 5.