

Answers to Worksheet 1 on Power Series

1. $e^6 + 2e^6(x-3) + \frac{4e^6(x-3)^2}{2!} + \frac{8e^6(x-3)^3}{3!} + \dots = \sum_{n=0}^{\infty} \frac{2^n e^6 (x-3)^n}{n!}$

2. $1 - (x-1) + (x-1)^2 - (x-1)^3 + \dots = \sum_{n=0}^{\infty} (-1)^n (x-1)^n$

3. $(x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4} + \dots = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} (x-1)^n}{n}$

4. $\frac{1}{2} + \frac{\sqrt{3}}{2} \left(x - \frac{\pi}{6}\right) - \frac{\left(x - \frac{\pi}{6}\right)^2}{2 \cdot 2!} - \frac{\sqrt{3} \left(x - \frac{\pi}{6}\right)^3}{2 \cdot 3!} + \dots$

5. $\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \left(x + \frac{\pi}{4}\right) - \frac{\sqrt{2} \left(x + \frac{\pi}{4}\right)^2}{2 \cdot 2!} - \frac{\sqrt{2} \left(x + \frac{\pi}{4}\right)^3}{2 \cdot 3!} + \dots$

6. $1 - \frac{x}{2} + \frac{x^2}{2^2 \cdot 2!} - \frac{x^3}{2^3 \cdot 3!} + \dots = \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{2^n n!}$

7. $x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \frac{x^{14}}{7!} + \dots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+2}}{(2n+1)!}$

8. $\frac{1}{x} - \frac{9x}{2!} + \frac{81x^3}{4!} - \frac{729x^5}{6!} + \dots = \sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n} x^{2n-1}}{(2n)!}$

9. $x^2 - x^3 + \frac{x^4}{2!} - \frac{x^5}{3!} + \dots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+2}}{n!}$

10. $\frac{2x^2}{2!} - \frac{8x^4}{4!} + \frac{32x^6}{6!} - \frac{128x^8}{8!} + \dots = \sum_{n=0}^{\infty} \frac{(-1)^{n+1} 2^{2n-1} x^{2n}}{(2n)!}$

11. 0.310

12. 0.905

13. (a) $1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - \dots$

(b) $1 + \frac{1}{2}x^3 - \frac{1}{8}x^6 + \frac{1}{16}x^9 - \dots$

(c) $4 + x + \frac{1}{8}x^4 - \frac{1}{56}x^7 + \frac{1}{160}x^{10} - \dots$

14. (a) $1 - (x-2) + (x-2)^2 - (x-2)^3 + \dots = \sum_{n=0}^{\infty} (-1)^n (x-2)^n$

(Integrate a) to get b)

(b) $(x-2) - \frac{(x-2)^2}{2} + \frac{(x-2)^3}{3} - \frac{(x-2)^4}{4} + \dots = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} (x-2)^n}{n}$

in it. concl.

(c) $\ln \frac{3}{2} \approx \frac{1}{2} - \frac{1}{8} = \frac{3}{8}$ by the Alternating Series Remainder $\left(|\text{Error}| < \frac{1}{24} < 0.05 \right)$

ln|x-1|
x=2
ln|2-1|=0
(2,0)
plug in to find c.
205
C = -2

x > 1/2