WORKSHEET 8: SERIES
POWER SERIES

Work the following on notebook paper. No Calculator except on 7(c).
On problems 1-5, find a power series for the given function, centered at the given value of a. Give the first four nonzero terms and the general term.

1. $f(x)=\dfrac{1}{1+x}$, $a=0$
2. $f(x)=\dfrac{1}{1+x^2}$, $a=0$
3. $f(x)=\dfrac{3}{x+2}$, $a=0$
4. $f(x)=\dfrac{x}{1-2x}$, $a=0$
5. $f(x)=\dfrac{1}{4-x}$, $a=1$

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6. Let f be the function given by $f(t)=\dfrac{4}{1+t^2}$ and G be the function given by $G(x)=\int_0^x f(t)dt$.
   (a) Find the first four nonzero terms and the general term for the power series expansion of $f(t)$ about $t=0$.
   (b) Find the first four nonzero terms and the general term of the power series expansion of $G(x)$ about x=0.
   (c) Find the interval of convergence of the power series in part (b). Justify your answer.

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7. Let f be the function given by $f(x)=e^{-2x^2}$
   (a) Find the first four nonzero terms and the general term of the power series for $f(x)$ about $x=0$.
   (b) Find the interval of convergence of the power series for $f(x)$ about $x=0$. Show the analysis that leads to your conclusion.
   (c) Let g be the function given by the sum of the first four nonzero terms of the power series for $f(x)$ about $x=0$. Show that $|f(x)-g(x)|<0.02$ for $-0.6\le x\le 0.6$.

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8. The Maclaurin series for $f(x)$ is given by $1+\dfrac{x}{2!}+\dfrac{x^2}{3!}+\dfrac{x^3}{4!}+\cdots+\dfrac{x^n}{(n+1)!}+\cdots$

(a) Find $f'(0)$ and $f^{(17)}(0)$.
(b) For what values of x does the given series converge? Show your reasoning.
(c) Let $g(x)=xf(x)$. Write the Maclaurin series for $g(x)$ in terms of a familiar function without using series. Then, write $f(x)$ in terms of the same familiar function.

By recognizing each series in problems 9-12 as a Taylor series evaluated at a particular value of x, find the sum of each of the following convergent series.

9. $1+\dfrac{2}{1!}+\dfrac{4}{2!}+\dfrac{8}{3!}+\cdots+\dfrac{2^n}{n!}+\cdots$
10. $1-\dfrac{1}{3!}+\dfrac{1}{5!}-\dfrac{1}{7!}+\cdots+\dfrac{(-1)^n}{(2n+1)!}+\cdots$
11. $1+\dfrac{1}{4}+\left(\dfrac{1}{4}\right)^2+\left(\dfrac{1}{4}\right)^3+\cdots+\left(\dfrac{1}{4}\right)^n+\cdots$
12. $1-\dfrac{100}{2!}+\dfrac{10{,}000}{4!}+\cdots+\dfrac{(-1)^n\cdot 10^{2n}}{(2n)!}+\cdots$