

Answers to Worksheet 2 on Power Series

1.  $1 - x + x^2 - x^3 + \dots = \sum_{n=0}^{\infty} (-1)^n x^n$

2.  $1 - x^2 + x^4 - x^6 + \dots = \sum_{n=0}^{\infty} (-1)^n x^{2n}$

3.  $\frac{3}{2} - \frac{3x}{4} + \frac{3x^2}{8} - \frac{3x^3}{16} + \dots = \frac{3}{2} \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{2^n}$

4.  $x + 2x^2 + 4x^3 + 8x^4 + \dots = \sum_{n=0}^{\infty} 2^n x^{n+1}$

5.  $\frac{1}{3} + \frac{x-1}{9} + \frac{(x-1)^2}{27} + \frac{(x-1)^3}{81} + \dots = \sum_{n=0}^{\infty} \frac{(x-1)^n}{3^{n+1}}$

6. (a)  $4 - 4t^2 + 4t^4 - 4t^6 + \dots = \sum_{n=0}^{\infty} 4(-1)^n t^{2n}$

(b)  $4x - \frac{4x^3}{3} + \frac{4x^5}{5} - \frac{4x^7}{7} + \dots = \sum_{n=0}^{\infty} \frac{4(-1)^n x^{2n+1}}{2n+1}$

(c)  $-1 \leq x \leq 1$

7. (a)  $1 - 2x^2 + \frac{(2x^2)^2}{2!} - \frac{(2x^2)^3}{3!} + \dots = \sum_{n=0}^{\infty} \frac{(-1)^n (2x^2)^n}{n!}$

(b) Converges for all  $x$

(c) Since  $f$  is a convergent alternating series whose terms decrease in magnitude,  $|f(x) - g(x)| < 0.0112 < 0.02$  by the Alternating Series Remainder.

8. (a)  $\frac{1}{2}; \frac{1}{18}$

(b) Converges for all  $x$

(c)  $x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots = \sum_{n=1}^{\infty} \frac{x^n}{n!}$

(d)  $g(x) = e^x - 1$

$$f(x) = \begin{cases} \frac{e^x - 1}{x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$$

9.  $e^2$

10.  $\sin 1$

11.  $\frac{4}{3}$

12.  $\cos 10$