CALCULUS BC
POWER SERIES WORKSHEET

Work the following on notebook paper.

1. (a) Find the fourth-degree Taylor polynomial for $\cos x$ about $x=0$. Then use your polynomial to approximate the value of $\cos 0.8$, and use Taylor's Theorem to determine the accuracy of the approximation. Give three decimal places.
   (b) Find an interval $[a, b]$ such that $a \le \cos(0.8) \le b$.
   (c) Could cos 0.8 equal 0.695? Show why or why not.
2. (a) Write the fourth-degree Maclaurin polynomial for $f(x)=e^x$. Then use your polynomial to approximate e^{-1}, and find a Lagrange error bound for the maximum error when $|x| \le 1$. Give three decimal places.
   (b) Find an interval $[a, b]$ such that $a \le e^{-1} \le b$.
3. Let f be a function that has derivatives of all orders for all real numbers x Assume that $f(5)=6,\ f'(5)=8,\ f''(5)=30,\ f'''(5)=48,$ and $\left|f^{(4)}(x)\right| \le 75$ for all x in the interval $[5, 5.2]$.
   (a) Find the third-degree Taylor polynomial about $x=5$ for $f(x)$.
   (b) Use your answer to part (a) to estimate the value of $f(5.2)$. What is the maximum possible error in making this estimate? Give three decimal places.
   (c) Find an interval $[a, b]$ such that $a \le f(5.2) \le b$. Give three decimal places.
   (d) Could $f(5.2)$ equal 8.254? Show why or why not.
   (d) Let $g(x)=x \cdot f(x^2)$. Find the Maclaurin series for $g(x)$. (Write as many nonzero terms as possible.)
   (e) Let $h(x)$ be a function that has the properties $h(0)=5$ and $h'(x)=f(x)$. Find the Maclaurin series for $h(x)$. (Write as many terms as possible.)

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4. Find the first four nonzero terms of the power series for $f(x)=\sin x$ centered at $x=\dfrac{3\pi}{4}$.

Find the first four nonzero terms and the general term for the Maclaurin series for:

5. $f(x)=x\cos(x^3)$

6. $g(x)=\dfrac{1}{1+x^2}$

Find the radius and interval of convergence for:

7. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n (x-2)^n}{3^n n^2}$

8. $\displaystyle\sum_{n=0}^{\infty} (2n)!(x-5)^n$

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9. Use the Maclaurin series for $\cos x$ to find $\lim_{x\to 0}\dfrac{1-\cos x}{x}$.

10. The Taylor series about $x=3$ for a certain function f converges to $f(x)$ for all x in the interval of convergence. The nth derivative of f at $x=3$ is given by

$$f^{(n)}(3)=\frac{(-1)^n n!}{5^n(n+3)} \text{ and } f(3)=\frac{1}{3}.$$

(a) Write the fourth-degree Taylor polynomial for f about $x=3$.
(b) Find the radius of convergence of the Taylor series for f about $x=3$.
(c) Show that the third-degree Taylor polynomial approximates $f(4)$ with an error less than $\dfrac{1}{4000}$.

Multiple Choice.

11. What are all the values of x for which the series $\sum_{n=1}^{\infty}\dfrac{x^n}{n}$ converges?

$(A)\ -1\le x\le 1$ $(B)\ -1<x<1$ $(C)\ -1<x\le 1$ $(D)\ -1\le x<1$ (E) All real x

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12. The coefficient of x^6 in the Taylor series expansion about $x=0$ for $f(x)=\sin(x^2)$ is

$(A)\ -\dfrac{1}{6}$ $(B)\ 0$ $(C)\ \dfrac{1}{120}$ $(D)\ \dfrac{1}{6}$ $(E)\ 1$

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13. If f is a function such that $f'(x)=\sin(x^2)$, then the coefficient of x^7 in the Taylor series for $f(x)$ about $x=0$ is

$(A)\ \dfrac{1}{7!}$ $(B)\ \dfrac{1}{7}$ $(C)\ 0$ $(D)\ -\dfrac{1}{42}$ $(E)\ -\dfrac{1}{7!}$