CALCULUS BC
WORKSHEET 6 ON SERIES

Work the following on notebook paper. Use may use your calculator on problems 11 and 12.

On problems 1 and 2, find the radius of convergence and the interval of convergence.

1. $\sum_{n=1}^{\infty} \frac{(x-5)^n}{n3^n}$

2. $\sum_{n=0}^{\infty} n!(x-2)^n$

Use Taylor series to evaluate.

3. $\lim_{x\to 0} \frac{1-\cos(2x)}{x^3}$

4. $\lim_{x\to 0} \frac{e^{3x}-1}{x}$

Find the sum.

5. $1+\frac{3}{1!}+\frac{9}{2!}+\frac{27}{3!}+\ldots+\frac{3^n}{n!}+\ldots$

6. $1-\frac{1}{3!}+\frac{1}{5!}-\frac{1}{7!}+\ldots+\frac{(-1)^n}{(2n+1)!}+\ldots$

7. $\frac{1}{5}-\frac{1}{25}+\frac{1}{125}-\frac{1}{625}+\ldots+\frac{(-1)^{n+1}}{5^n}+\ldots$

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8. A function f is defined by $f(x)=\frac{1}{4}+\frac{2}{4^2}x+\frac{3}{4^3}x^2+\ldots+\frac{n+1}{4^{n+1}}x^n+\ldots$

for all x in the interval of convergence of the power series.

(a) Find the interval of convergence for this power series. Show the work that leads to your answer.

(b) Find $\lim_{x\to 0} \frac{f(x)-\frac{1}{4}}{x}$.

(c) Write the first three nonzero terms and the general term for an infinite series that represents $\int_0^1 f(x)dx$.

(d) Find the sum of the series determined in part (c).

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9. Let f be the function given by $f(x)=\ln(3-x)$. Find the third-degree Taylor polynomial for f about $x = 2$.

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10. If f is a function such that $f'(x)=\cos\left(x^3\right)$, find the coefficient of the x^7 term in the Taylor polynomial for $f(x)$ about $x = 0$.

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11. The graph of the function represented by the Maclaurin series

$1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+\ldots+\frac{(-1)^n x^n}{n!}+\ldots$ intersects the graph of $y=x^3$ at $x = ?$

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12. The function f has derivatives of all orders for all real numbers x. Assume $f(5) = 2$, $f'(5) = -3$, $f''(5) = -1$, and $f'''(5) = 6$.

(a) Write the third-degree Taylor polynomial for f about $x = 5$, and use it to approximate $f(4.2)$.

(b) The fourth derivative of f satisfies the inequality $\left|f^{(4)}(x)\right| \le 2$ for all x in the closed interval [4,2, 5]. Use the Lagrange error bound on the approximation of $f(4.2)$ to explain why $f(4.2) \ne 3.4$.

(c) Write the fourth-degree Taylor polynomial, $P(x)$, for $g(x) = f\left(x^2 + 5\right)$ about $x = 0$. Use P to explain whether g has a relative maximum, relative minimum, or neither a relative maximum nor a relative minimum at $x = 0$.

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13. The Maclaurin series for the function f is given by $f(x) = \sum_{n=1}^{\infty} \frac{(3x)^n}{n}$.

(a) Write the first four nonzero terms of the Maclaurin series for $f(x)$.

(b) Write the first four nonzero terms and the general term for the Maclaurin series for $f'(x)$.

(c) Use your answer to (b) to find the value of $f'\left(-\frac{1}{4}\right)$.

(d) Given that $g'(x) = f(x)$ and that $g(0) = 1$, write the first four terms and the general term for $g(x)$.