

WORSHEET 4: SERIES

1. (98 BC: No Calc) Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{n}{n+2}$

II. $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n}$

III. $\sum_{n=1}^{\infty} \frac{1}{n}$

- (A) None
- (B) II only
- (C) III only
- (D) I and II only
- (E) I and III only

2. (98 BC: No Calc) If $\lim_{b \rightarrow \infty} \int_1^b \frac{dx}{x^p}$ is finite, then which of the following must be true?

(A) $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges

(B) $\sum_{n=1}^{\infty} \frac{1}{n^p}$ diverges

(C) $\sum_{n=1}^{\infty} \frac{1}{n^{p-2}}$ converges

(D) $\sum_{n=1}^{\infty} \frac{1}{n^{p-1}}$ converges

(E) $\sum_{n=1}^{\infty} \frac{1}{n^{p+1}}$ diverges

3. (98 BC: No Calc) $\lim_{x \rightarrow 1} \frac{\int_1^x e^{t^2} dt}{x^2 - 1}$ is

(A) 0

(B) 1

(C) $\frac{e}{2}$

(D) e

(E) nonexistent

4. (98 BC: Calc OK) For what integer $k, k > 1$, will both $\sum_{n=1}^{\infty} \frac{(-1)^{kn}}{n}$ and $\sum_{n=1}^{\infty} \left(\frac{k}{4}\right)^n$ converge?

(A) 6

(B) 5

(C) 4

(D) 3

(E) 2

5. REVIEW: $\frac{dy}{dx} = \sqrt{1-y^2}$, then $\frac{d^2y}{dx^2} =$

(A) $-2y$

(B) $-y$

(C) $\frac{-y}{\sqrt{1-y^2}}$

(D) y

(E) $\frac{1}{2}$

5. Find a formula for the n th term of the sequence. (Assume that n begins with 1.)

$$\frac{2}{1}, \frac{4}{1}, \frac{6}{2}, \frac{8}{6}, \frac{10}{24}, \dots$$

then determine if the sequence converges.

6. Determine if $\sum_{n=1}^{\infty} 2(-0.9)^n$ converges or diverges. If it converges, find the sum.

7. Determine the convergence of the following series: (this is NOT a multiple choice question).

(A) $\sum_{n=0}^{\infty} \frac{1}{2^n}$

(B) $\sum_{n=1}^{\infty} \frac{1}{n(n+2)}$

$$(C) \sum_{n=1}^{\infty} \frac{1}{n^2}$$

$$(D) \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$$

8. Which of the above series (in problem 7) is a telescoping series? What is its sum????

Determine if the following converge or diverge. Clearly state the test and any info about comparisons.

$$9. \sum_{n=1}^{\infty} \frac{1}{n^2} \cos\left(\frac{1}{n}\right)$$

$$10. \sum_{n=1}^{\infty} n e^{-n}$$

$$11. \sum_{n=1}^{\infty} \frac{1}{n^2 + 1}$$

$$12. \sum_{n=1}^{\infty} \frac{4n}{(n+1)(n+2)}$$

$$13. \sum_{n=1}^{\infty} \frac{1}{3^n + 1}$$

$$14. \sum_{n=1}^{\infty} \frac{1}{n^2 + 3}$$

$$15. \sum_{n=1}^{\infty} \frac{6n^2 - 6}{n^3 - n}$$

$$16. \sum_{n=1}^{\infty} \frac{n}{\sqrt{n^3 + 2n}}$$

$$17. \sum_{n=1}^{\infty} \cos(3n\pi)$$

$$18. \sum_{n=1}^{\infty} \frac{n!}{(n+5)!}$$

$$19. \sum_{n=1}^{\infty} \frac{n!}{3^n}$$

$$20. \sum_{n=1}^{\infty} \frac{2^n}{(n+6)!}$$

$$21. \sum_{n=1}^{\infty} \frac{n^8}{8^n}$$

$$22. \sum_{n=1}^{\infty} \frac{n+1}{3n+1}$$

$$23. \sum_{n=1}^{\infty} \left(\frac{2n-1}{3n+5}\right)^n$$

$$24. \sum_{n=1}^{\infty} \frac{1}{1+e^{-n}}$$

Classify any of the following convergent series as absolutely or conditionally convergent. (18-22)

$$25. \sum_{n=1}^{\infty} \frac{(-1)^n (n-1)}{n\sqrt{n}}$$

$$26. \sum_{n=0}^{\infty} (-1)^n e^{-n}$$

$$27. \sum_{n=2}^{\infty} (-1)^n \frac{\ln n}{n}$$

$$28. \sum_{n=1}^{\infty} \left(-\frac{\pi}{e}\right)^n$$

$$29. \sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n}}{n+3}$$

30. Consider the series: $\sum_{n=1}^{\infty} \frac{(-1)^n}{2^n}$

- Show that the series is absolutely convergent.
- Calculate the sum of the first six terms. Round your answer to three decimal places.
- Find the number of terms necessary to approximate the sum of the series with an error less than 0.001