CALCULUS BC
WORKSHEET 7 ON POWER SERIES

Work the following on notebook paper. Use may use your calculator on problems 11 and 12.

On problems 1-3, find a Taylor series for $f(x)$ centered at the given value of a. Give the first four nonzero terms and the general term for each series.

1. $f(x) = e^{2x}$, $a = 3$

2. $f(x) = \dfrac{1}{x}$, $a = 1$

3. $f(x) = \ln x$, $a = 1$

On problems 4-5, find a Taylor series for $f(x)$ centered at the given value of a. Give the first four nonzero terms.

4. $f(x) = \sin x$, $a = \dfrac{\pi}{6}$

5. $f(x) = \cos x$, $a = -\dfrac{\pi}{4}$

On problems 6-10, find a Maclaurin series for $f(x)$. Give the first four nonzero terms and the general term for each series.

6. $f(x) = e^{-\frac{x}{2}}$

7. $f(x) = \sin(x^2)$

8. $f(x) = \dfrac{\cos(3x)}{x}$

9. $f(x) = x^2 e^{-x}$

10. $f(x) = \sin^2 x$ (HINT: use the power-reducing identity)

11. Use your answer for problem 7 to approximate $\int_0^1 \sin(x^2)\,dx$ correct to three decimal places.

12. Use series to approximate $\int_0^1 \cos(x^2)\,dx$ correct to three decimal places.

13. (a) Find the first four nonzero terms in the Taylor series expansion about $x = 0$ for $f(x) = \sqrt{1+x}$.

   (b) Use the results found in part (a) to find the first four nonzero terms in the Taylor series expansion about $x = 0$ for $g(x) = \sqrt{1+x^3}$.

   (c) Find the first four nonzero terms in the Taylor series expansion about $x = 0$ for the function h such that $h'(x) = \sqrt{1+x^3}$ and $h(0) = 4$.

14. Let f be the function defined by $f(x) = \dfrac{1}{x-1}$.

    (a) Write the first four terms and the general term of the Taylor series expansion of $f(x)$ about $x = 2$.

    (b) Use the result from part (a) to find the first four terms and the general term of the series expansion about $x = 2$ for $\ln|x-1|$.

    (c) Use the series in part (b) to compute a number that differs from $\ln\left(\dfrac{3}{2}\right)$ by less than 0.05. Justify your answer.