BC Review 17 Calculator Permitted
Do all work on separate notebook paper

_____ 1.

$\lim_{h\to 0}\dfrac{\ln(e+h)-1}{h}$ is

(A) $f'(e)$, where $f(x)=\ln x$

(B) $f'(e)$, where $f(x)=\dfrac{\ln x}{x}$

(C) $f'(1)$, where $f(x)=\ln x$

(D) $f'(1)$, where $f(x)=\ln(x+e)$

(E) $f'(0)$, where $f(x)=\ln x$

_____ 2.

The position of an object attached to a spring is given by $y(t)=\dfrac{1}{6}\cos(5t)-\dfrac{1}{4}\sin(5t)$, where t is time in seconds. In the first 4 seconds, how many times is the velocity of the object equal to 0?

(A) Zero
(B) Three
(C) Five
(D) Six
(E) Seven

_____ 3.

Let f be the function given by $f(x)=\cos(2x)+\ln(3x)$. What is the least value of x at which the graph of f changes concavity?

(A) 0.56 (B) 0.93 (C) 1.18 (D) 2.38 (E) 2.44

_____ 4.

If $0\le x\le 4$, of the following, which is the greatest value of x such that $\int_0^x (t^2-2t)\,dt\ge\int_2^x t\,dt$?

(A) 1.35 (B) 1.38 (C) 1.41 (D) 1.48 (E) 1.59

_____ 5.

If $\frac{dy}{dx} = (1+\ln x)\, y$ and if $y = 1$ when $x = 1$, then $y =$

(A) $e^{\frac{x^2-1}{x^2}}$

(B) $1+\ln x$

(C) $\ln x$

(D) $e^{2x+x\ln x-2}$

(E) $e^{x\ln x}$

_____ 6.

Let f be a continuous function on the closed interval $[-3,6]$. If $f(-3) = -1$ and $f(6) = 3$, then the Intermediate Value Theorem guarantees that

(A) $f(0) = 0$

(B) $f'(c) = \frac{4}{9}$ for at least one c between –3 and 6

(C) $-1 \le f(x) \le 3$ for all x between –3 and 6

(D) $f(c) = 1$ for at least one c between –3 and 6

(E) $f(c) = 0$ for at least one c between –1 and 3

_____ 7.

$\int x^2 \sin x\, dx =$

(A) $-x^2\cos x - 2x\sin x - 2\cos x + C$

(B) $-x^2\cos x + 2x\sin x - 2\cos x + C$

(C) $-x^2\cos x + 2x\sin x + 2\cos x + C$

(D) $-\frac{x^3}{3}\cos x + C$

(E) $2x\cos x + C$

_____ 8.

Let f be a twice differentiable function such that $f(1) = 2$ and $f(3) = 7$. Which of the following must be true for the function f on the interval $1 \le x \le 3$?

I. The average rate of change of f is $\frac{5}{2}$.

II. The average value of f is $\frac{9}{2}$.

III. The average value of f' is $\frac{5}{2}$.

(A) None
(B) I only
(C) III only
(D) I and III only
(E) II and III only

_____ 9.

$$\int \frac{dx}{(x-1)(x+3)} =$$

(A) $\frac{1}{4}\ln\left|\frac{x-1}{x+3}\right| + C$

(B) $\frac{1}{4}\ln\left|\frac{x+3}{x-1}\right| + C$

(C) $\frac{1}{2}\ln\left|(x-1)(x+3)\right| + C$

(D) $\frac{1}{2}\ln\left|\frac{2x+2}{(x-1)(x+3)}\right| + C$

(E) $\ln\left|(x-1)(x+3)\right| + C$

_____ 10.

The base of a solid is the region in the first quadrant enclosed by the graph of $y = 2 - x^2$ and the coordinate axes. If every cross section of the solid perpendicular to the y-axis is a square, the volume of the solid is given by

(A) $\pi\int_0^2 (2-y)^2\,dy$

(B) $\int_0^2 (2-y)\,dy$

(C) $\pi\int_0^{\sqrt{2}} \left(2-x^2\right)^2\,dx$

(D) $\int_0^{\sqrt{2}} \left(2-x^2\right)^2\,dx$

(E) $\int_0^{\sqrt{2}} \left(2-x^2\right)\,dx$

Free Response

11. 2008—AB/BC2B (Calculator Permitted)

For time $t \geq 0$ hours, let $r(t) = 120\left(1 - e^{-10t^2}\right)$ represent the speed, in kilometers per hour, at which a car travels along a straight road. The number of liters of gasoline used by the car to travel x kilometers is modeled by $g(x) = 0.05x\left(1 - e^{-x/2}\right)$.

(a) How many kilometers does the car travel during the first 2 hours?

(b) Find the rate of change with respect to time of the number of liters of gasoline used by the car when $t = 2$ hours. Indicate units of measure.

(c) How many liters of gasoline have been used by the car when it reaches a speed of 80 kilometers per hour?

12. 2008—BC6B (No Calculator)

Let f be the function given by $f(x) = \dfrac{2x}{1+x^2}$.

(a) Write the first four nonzero terms and the general term of the Taylor series for f about $x = 0$.

(b) Does the series found in part (a), when evaluated at $x = 1$, converge to $f(1)$? Explain why or why not.

(c) The derivative of $\ln\left(1+x^2\right)$ is $\dfrac{2x}{1+x^2}$. Write the first four nonzero terms of the Taylor series for $\ln\left(1+x^2\right)$ about $x = 0$.

(d) Use the series found in part (c) to find a rational number A such that $\left|A - \ln\left(\frac{5}{4}\right)\right| < \dfrac{1}{100}$. Justify your answer.