

- NC 21. Which of the following is equal to the area of the region inside the polar curve  $r = 2 \cos \theta$  and outside the polar curve  $r = \cos \theta$ ?

(A)  $3 \int_0^{\frac{\pi}{2}} \cos^2 \theta d\theta$  (B)  $3 \int_0^{\pi} \cos^2 \theta d\theta$  (C)  $\frac{3}{2} \int_0^{\frac{\pi}{2}} \cos^2 \theta d\theta$  (D)  $3 \int_0^{\frac{\pi}{2}} \cos \theta d\theta$  (E)  $3 \int_0^{\pi} \cos \theta d\theta$

From 1999 Acorn Book

- NC 23. The area of one loop of the graph of the polar equation  $r = 2 \sin(3\theta)$  is given by which of the following expressions?

(A)  $4 \int_0^{\frac{\pi}{3}} \sin^2(3\theta) d\theta$

(B)  $2 \int_0^{\frac{\pi}{3}} \sin(3\theta) d\theta$

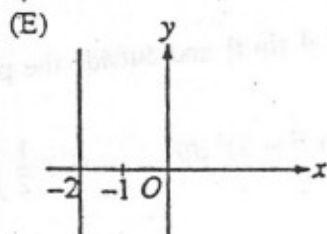
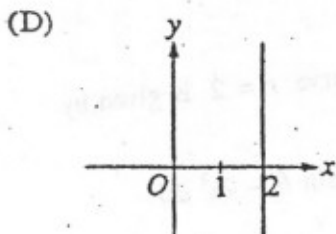
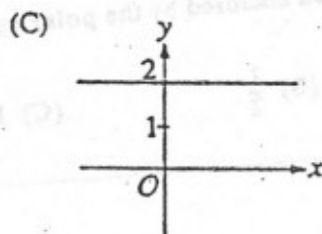
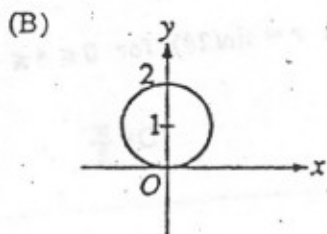
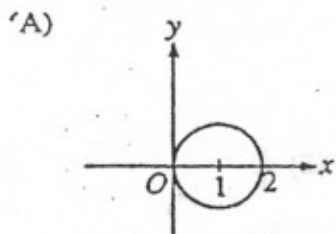
(C)  $2 \int_0^{\frac{\pi}{3}} \sin^2(3\theta) d\theta$

(D)  $2 \int_0^{\frac{2\pi}{3}} \sin^2(3\theta) d\theta$

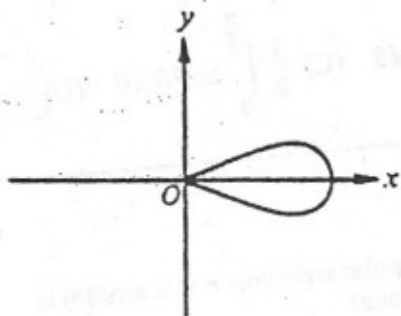
(E)  $2 \int_0^{\frac{2\pi}{3}} \sin(3\theta) d\theta$

From 1993 BC (1998 → ruled no more strictly pre-cal)

- NC 5. Which of the following represents the graph of the polar curve  $r = 2 \sec \theta$ ?



From 1988 BC



NC23. Which of the following gives the area of the region enclosed by the loop of the graph of the polar curve  $r = 4 \cos(3\theta)$  shown in the figure above?

(A)  $16 \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \cos(3\theta) d\theta$

(B)  $8 \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \cos(3\theta) d\theta$

(C)  $8 \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \cos^2(3\theta) d\theta$

(D)  $16 \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \cos^2(3\theta) d\theta$

(E)  $8 \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \cos^2(3\theta) d\theta$

From 1985 BC

NC The area of the region enclosed by the polar curve  $r = \sin(2\theta)$  for  $0 \leq \theta \leq \frac{\pi}{2}$  is

(A) 0

(B)  $\frac{1}{2}$

(C) 1

(D)  $\frac{\pi}{8}$

(E)  $\frac{\pi}{4}$

From 1998 BC

NC 19. The area of the region inside the polar curve  $r = 4 \sin \theta$  and outside the polar curve  $r = 2$  is given by

(A)  $\frac{1}{2} \int_0^{\pi} (4 \sin \theta - 2)^2 d\theta$

(B)  $\frac{1}{2} \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} (4 \sin \theta - 2)^2 d\theta$

(C)  $\frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (4 \sin \theta - 2)^2 d\theta$

(D)  $\frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (16 \sin^2 \theta - 4) d\theta$

(E)  $\frac{1}{2} \int_0^{\pi} (16 \sin^2 \theta - 4) d\theta$

Answers to Polars

1997

21. A

1999

23. C

1993

5. D

1988

23. E

1985

24. D

1998

19. D



$$\pi - \int_0^{\pi} (3 \cos 3\theta)^2 d\theta = \int_0^{\pi} 9 \cos^2 3\theta d\theta = 9 \int_0^{\pi} \frac{1 + \cos 6\theta}{2} d\theta = \frac{9}{2} \left[ \theta + \frac{\sin 6\theta}{6} \right]_0^{\pi} = \frac{9}{2} \pi$$

$$\pi = \dots = 0.636 \dots \int_0^{\pi} \frac{1}{2} d\theta$$

$$\pi = \dots = 0.605 \dots \int_0^{\pi} \frac{1}{2} d\theta$$

$$x = 2 \sin 3\theta \cos 3\theta = x$$

$$y = 2 \sin 3\theta \sin 3\theta = y$$

$$2 \sin 3\theta \cos 3\theta + 2 \sin 3\theta \sin 3\theta = 2 \sin 6\theta$$

$$2 \sin 3\theta \cos 3\theta + 2 \cos 3\theta \sin 3\theta = 4 \sin 6\theta$$

$$r = 2 \sin 6\theta \implies \theta = \frac{1}{6} \arcsin \frac{r}{2}$$

$$\frac{dr}{d\theta} = 2 \cos 6\theta$$

$$(x^2 + y^2)^2 = 4r^2 \implies (x^2 + y^2)^2 = 4(2 \sin 6\theta)^2 = 16 \sin^2 6\theta$$

$$2(x^2 + y^2)(2x dx + 2y dy) = 32 \sin 6\theta \cos 6\theta d\theta = 32 \sin 12\theta d\theta$$

$$2(x^2 + y^2) dx + 2(x^2 + y^2) dy = 16 \sin 12\theta d\theta$$