AP® CALCULUS BC
2006 SCORING GUIDELINES

Question 3

An object moving along a curve in the xy-plane is at position $(x(t), y(t))$ at time t, where

$$\frac{dx}{dt} = \sin^{-1}\left(1 - 2e^{-t}\right) \text{ and } \frac{dy}{dt} = \frac{4t}{1 + t^3}$$

for $t \geq 0$. At time $t = 2$, the object is at the point $(6, -3)$. (Note: $\sin^{-1} x = \arcsin x$)

(a) Find the acceleration vector and the speed of the object at time $t = 2$.

(b) The curve has a vertical tangent line at one point. At what time t is the object at this point?

(c) Let $m(t)$ denote the slope of the line tangent to the curve at the point $(x(t), y(t))$. Write an expression for $m(t)$ in terms of t and use it to evaluate $\lim_{t\to\infty} m(t)$.

(d) The graph of the curve has a horizontal asymptote $y = c$. Write, but do not evaluate, an expression involving an improper integral that represents this value c.

(a) $a(2) = \langle 0.395 \text{ or } 0.396, -0.741 \text{ or } -0.740 \rangle$ Speed $= \sqrt{x'(2)^2 + y'(2)^2} = 1.207$ or 1.208	$2: \begin{cases} 1: \text{acceleration} \\ 1: \text{speed} \end{cases}$
(b) $\sin^{-1}\left(1 - 2e^{-t}\right) = 0$ $1 - 2e^{-t} = 0$ $t = \ln 2 = 0.693$ and $\frac{dy}{dt} \neq 0$ when $t = \ln 2$	$2: \begin{cases} 1: x'(t) = 0 \\ 1: \text{answer} \end{cases}$
(c) $m(t) = \frac{4t}{1 + t^3} \cdot \frac{1}{\sin^{-1}\left(1 - 2e^{-t}\right)}$ $\lim_{t\to\infty} m(t) = \lim_{t\to\infty}\left(\frac{4t}{1 + t^3} \cdot \frac{1}{\sin^{-1}\left(1 - 2e^{-t}\right)}\right)$ $= 0\left(\frac{1}{\sin^{-1}(1)}\right) = 0$	$2: \begin{cases} 1: m(t) \\ 1: \text{limit value} \end{cases}$
(d) Since $\lim_{t\to\infty} x(t) = \infty$, $c = \lim_{t\to\infty} y(t) = -3 + \int_2^{\infty} \frac{4t}{1 + t^3}\, dt$	$3: \begin{cases} 1: \text{integrand} \\ 1: \text{limits} \\ 1: \text{initial value consistent with lower limit} \end{cases}$

Question 2

An object moving along a curve in the xy-plane is at position $(x(t), y(t))$ at time t, where

$$\frac{dx}{dt} = \tan\left(e^{-t}\right) \text{ and } \frac{dy}{dt} = \sec\left(e^{-t}\right)$$

for $t \geq 0$. At time $t = 1$, the object is at position $(2, -3)$.

(a) Write an equation for the line tangent to the curve at position $(2, -3)$.

(b) Find the acceleration vector and the speed of the object at time $t = 1$.

(c) Find the total distance traveled by the object over the time interval $1 \leq t \leq 2$.

(d) Is there a time $t \geq 0$ at which the object is on the y-axis? Explain why or why not.

(a) $$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\sec\left(e^{-t}\right)}{\tan\left(e^{-t}\right)} = \frac{1}{\sin\left(e^{-t}\right)}$$

$$\left.\frac{dy}{dx}\right|_{(2,-3)} = \frac{1}{\sin\left(e^{-1}\right)} = 2.780 \text{ or } 2.781$$

$$y + 3 = \frac{1}{\sin\left(e^{-1}\right)}(x - 2)$$

$2 : \begin{cases} 1 : \left.\frac{dy}{dx}\right|_{(2,-3)} \\ 1 : \text{equation of tangent line} \end{cases}$

(b) $x''(1) = -0.42253,\ y''(1) = -0.15196$

$a(1) = \langle -0.423, -0.152 \rangle$ or $\langle -0.422, -0.151 \rangle$.

$$\text{speed} = \sqrt{\left(\sec\left(e^{-1}\right)\right)^2 + \left(\tan\left(e^{-1}\right)\right)^2} = 1.138 \text{ or } 1.139$$

$2 : \begin{cases} 1 : \text{acceleration vector} \\ 1 : \text{speed} \end{cases}$

(c) $$\int_1^2 \sqrt{(x'(t))^2 + (y'(t))^2}\, dt = 1.059$$

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(d) $$x(0) = x(1) - \int_0^1 x'(t)\, dt = 2 - 0.775553 > 0$$

The particle starts to the right of the y-axis.
Since $x'(t) > 0$ for all $t \geq 0$, the object is always moving to the right and thus is never on the y-axis.

$3 : \begin{cases} 1 : x(0) \text{ expression} \\ 1 : x'(t) > 0 \\ 1 : \text{conclusion and reason} \end{cases}$

AP® CALCULUS BC
2007 SCORING GUIDELINES (Form B)

Question 2

An object moving along a curve in the xy-plane is at position $(x(t), y(t))$ at time t with

$$\frac{dx}{dt} = \arctan\left(\frac{t}{1+t}\right) \text{ and } \frac{dy}{dt} = \ln\left(t^2+1\right)$$

for $t \geq 0$. At time $t = 0$, the object is at position $(-3, -4)$. (Note: $\tan^{-1} x = \arctan x$)

(a) Find the speed of the object at time $t = 4$.

(b) Find the total distance traveled by the object over the time interval $0 \leq t \leq 4$.

(c) Find $x(4)$.

(d) For $t > 0$, there is a point on the curve where the line tangent to the curve has slope 2. At what time t is the object at this point? Find the acceleration vector at this point.

(a) Speed $= \sqrt{x'(4)^2 + y'(4)^2} = 2.912$	1 : speed at $t = 4$
(b) Distance $= \int_0^4 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = 6.423$	2 : { 1 : integral 1 : answer
(c) $x(4) = x(0) + \int_0^4 x'(t)\,dt$ $= -3 + 2.10794 = -0.892$	3 : { 2 : { 1 : integrand 1 : uses $x(0) = -3$ 1 : answer
(d) The slope is 2, so $\dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = 2$, or $\ln\left(t^2+1\right) = 2\arctan\left(\frac{t}{1+t}\right)$. Since $t > 0$, $t = 1.35766$. At this time, the acceleration is $\langle x''(t), y''(t)\rangle\big\vert_{t=1.35766} = \langle 0.135, 0.955\rangle$.	3 : { 1 : $\dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = 2$ 1 : t-value 1 : values for x'' and y''