An object moving along a curve in the xy-plane has position $(x(t), y(t))$ at time t with

$$\frac{dx}{dt} = \cos(t^3) \text{ and } \frac{dy}{dt} = 3\sin(t^2)$$

for $0 \le t \le 3$. At time $t = 2$, the object is at position (4,5).

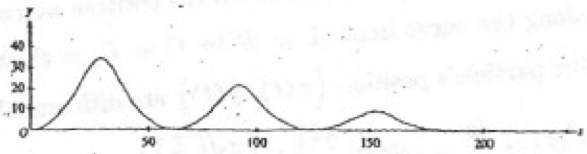
(a) Write an equation for the line tangent to the curve at (4,5).

(b) Find the speed of the object at time $t = 2$.

(c) Find the total distance traveled by the object over the time interval $0 \le t \le 1$.

(d) Find the position of the object at time $t = 3$.

(a) $\frac{dy}{dx} = \frac{3\sin(t^2)}{\cos(t^3)}$ $\left.\frac{dy}{dx}\right\|_{t=2} = \frac{3\sin(2^2)}{\cos(2^3)} = 15.604$ $y - 5 = 15.604(x - 4)$	1 : tangent line
(b) Speed = $\sqrt{\cos^2(8) + 9\sin^2(4)} = 2.275$	1 : answer
(c) Distance = $\int_0^1 \sqrt{\cos^2(t^3) + 9\sin^2(t^2)}\, dt$ $= 1.458$	3 : { 2 : distance integral < −1 > each integrand error < −1 > error in limits 1 : answer
(d) $x(3) = 4 + \int_2^3 \cos(t^3)\,dt = 3.953$ or 3.954 $y(3) = 5 + \int_2^3 3\sin(t^2)\,dt = 4.906$	4 : { 1 : definite integral for x 1 : answer for $x(3)$ 1 : definite integral for y 1 : answer for $y(3)$



The figure above shows the path traveled by a roller coaster car over the time interval $0 \le t \le 18$ seconds. The position of the car at time t seconds can be modeled parametrically by $x(t) = 10t + 4\sin t$, $y(t) = (20 - t)(1 - \cos t)$, where x and y are measured in meters. The derivatives of these functions are given by

$x'(t) = 10 + 4\cos t$, $y'(t) = (20 - t)\sin t + \cos t - 1$.

(a) Find the slope of the path at time $t = 2$. Show the computations that lead to your answer.

(b) Find the acceleration vector of the car at the time when the car's horizontal position is $x = 140$.

(c) Find the time t at which the car is at its maximum height, and find the speed, in m/sec, of the car at this time.

(d) For $0 < t < 18$, there are two times at which the car is at ground level ($y = 0$). Find these two times and write an expression that gives the average speed, in m/sec, of the car between these two times. Do not evaluate the expression.

(a) Slope $= \left.\dfrac{dy}{dx}\right|_{t=2} = \dfrac{y'(2)}{x'(2)} = \dfrac{18\sin 2 + \cos 2 - 1}{10 + 4\cos 2}$

$= 1.793$ or 1.794

1 : answer using $\dfrac{dy}{dx} = \dfrac{dy}{dt} \Big/ \dfrac{dx}{dt}$

(b) $x(t) = 10t + 4\sin t = 140$; $t_0 = 13.647083$

$x''(t_0) = -3.529$, $y''(t_0) = 1.225$ or 1.226

Acceleration vector is $\langle -3.529, 1.225 \rangle$ or $\langle -3.529, 1.226 \rangle$

2 { 1 : identifies acceleration vector as derivative of velocity vector
1 : computes acceleration vector when $x = 140$ }

(c) $y'(t) = (20 - t)\sin t + \cos t - 1 = 0$

$t_1 = 3.023$ or 3.024 at maximum height

Speed $= \sqrt{\left(x'(t_1)\right)^2 + \left(y'(t_1)\right)^2} = |x'(t_1)|$

$= 6.027$ or 6.028

3 { 1 : sets $y'(t) = 0$
1 : selects first $t > 0$
1 : speed }

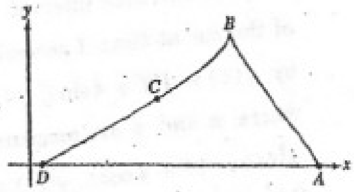
(d) $y(t) = 0$ when $t = 2\pi$ and $t = 4\pi$

Average speed $= \dfrac{1}{2\pi}\displaystyle\int_{2\pi}^{4\pi} \sqrt{\left(x'(t)\right)^2 + \left(y'(t)\right)^2}\,dt$

$= \dfrac{1}{2\pi}\displaystyle\int_{2\pi}^{4\pi} \sqrt{(10 + 4\cos t)^2 + ((20 - t)\sin t + \cos t - 1)^2}\,dt$

3 { 1 : $t = 2\pi, t = 4\pi$
1 : limits and constant
1 : integrand }

A particle starts at point A on the positive x-axis at time $t = 0$ and travels along the curve from A to B to C to D, as shown above. The coordinates of the particle's position $\left(x(t), y(t)\right)$ are differentiable functions of t, where



$$x'(t) = \frac{dx}{dt} = -9\cos\left(\frac{\pi t}{6}\right)\sin\left(\frac{\pi\sqrt{t+1}}{2}\right) \text{ and } y'(t) = \frac{dy}{dt} \text{ is not explicitly given.}$$

At time $t = 9$, the particle reaches its final position at point D on the positive x-axis.

(a) At point C, is $\frac{dy}{dt}$ positive? At point C, is $\frac{dx}{dt}$ positive? Give a reason for each answer.

(b) The slope of the curve is undefined at point B. At what time t is the particle at point B?

(c) The line tangent to the curve at the point $\left(x(8), y(8)\right)$ has equation $y = \frac{5}{9}x - 2$. Find the velocity vector and the speed of the particle at this point.

(d) How far apart are points A and D, the initial and final positions, respectively, of the particle?

(a) At point C, $\frac{dy}{dt}$ is not positive because $y(t)$ is decreasing along the arc BD as t increases.

At point C, $\frac{dx}{dt}$ is not positive because $x(t)$ is decreasing along the arc BD as t increases.

$2: \begin{cases} 1: \frac{dy}{dt} \text{ not positive with reason} \\ 1: \frac{dx}{dt} \text{ not positive with reason} \end{cases}$

(b) $\frac{dx}{dt} = 0$; $\cos\left(\frac{\pi t}{6}\right) = 0$ or $\sin\left(\frac{\pi\sqrt{t+1}}{2}\right) = 0$

$\frac{\pi t}{6} = \frac{\pi}{2}$ or $\frac{\pi\sqrt{t+1}}{2} = \pi$; $t = 3$ for both.

Particle is at point B at $t = 3$.

$2: \begin{cases} 1: \text{sets } \frac{dx}{dt} = 0 \\ 1: t = 3 \end{cases}$

(c) $x'(8) = -9\cos\left(\frac{4\pi}{3}\right)\sin\left(\frac{3\pi}{2}\right) = -\frac{9}{2}$

$\frac{y'(8)}{x'(8)} = \frac{dy}{dx} = \frac{5}{9}$

$y'(8) = \frac{5}{9}x'(8) = -\frac{5}{2}$

The velocity vector is $< -4.5, -2.5 >$.

Speed $= \sqrt{4.5^2 + 2.5^2} = 5.147$ or 5.148

$3: \begin{cases} 1: x'(8) \\ 1: y'(8) \\ 1: \text{speed} \end{cases}$

(d) $x(9) - x(0) = \int_0^9 x'(t)\,dt$

$= -39.255$

The initial and final positions are 39.255 apart.

$2: \begin{cases} 1: \text{integral} \\ 1: \text{answer} \end{cases}$

An object moving along a curve in the xy-plane has position $(x(t), y(t))$ at time $t \geq 0$ with $\frac{dx}{dt} = 3 + \cos(t^2)$. The derivative $\frac{dy}{dt}$ is not explicitly given. At time $t = 2$, the object is at position $(1, 8)$.

(a) Find the x-coordinate of the position of the object at time $t = 4$.

(b) At time $t = 2$, the value of $\frac{dy}{dt}$ is -7. Write an equation for the line tangent to the curve at the point $(x(2), y(2))$.

(c) Find the speed of the object at time $t = 2$.

(d) For $t \geq 3$, the line tangent to the curve at $(x(t), y(t))$ has a slope of $2t + 1$. Find the acceleration vector of the object at time $t = 4$.

(a) $x(4) = x(2) + \int_2^4 \left(3 + \cos(t^2)\right) dt$

$= 1 + \int_2^4 \left(3 + \cos(t^2)\right) dt = 7.132 \text{ or } 7.133$

$3: \begin{cases} 1 : \int_2^4 \left(3 + \cos(t^2)\right) dt \\ 1 : \text{handles initial condition} \\ 1 : \text{answer} \end{cases}$

(b) $\left.\frac{dy}{dx}\right|_{t=2} = \left.\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right|_{t=2} = \frac{-7}{3 + \cos 4} = -2.983$

$y - 8 = -2.983(x - 1)$

$2: \begin{cases} 1 : \text{finds } \left.\frac{dy}{dx}\right|_{t=2} \\ 1 : \text{equation} \end{cases}$

(c) The speed of the object at time $t = 2$ is $\sqrt{(x'(2))^2 + (y'(2))^2} = 7.382 \text{ or } 7.383$

1 : answer

(d) $x''(4) = 2.303$

$y'(t) = \frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} = (2t + 1)\left(3 + \cos(t^2)\right)$

$y''(4) = 24.813 \text{ or } 24.814$

The acceleration vector at $t = 4$ is $\langle 2.303, 24.813 \rangle$ or $\langle 2.303, 24.814 \rangle$.

$3: \begin{cases} 1 : x''(4) \\ 1 : \frac{dy}{dt} \\ 1 : \text{answer} \end{cases}$

AP® CALCULUS BC
2004 SCORING GUIDELINES (Form B)

Question 1

A particle moving along a curve in the plane has position $(x(t), y(t))$ at time t, where

$$\frac{dx}{dt} = \sqrt{t^4 + 9} \text{ and } \frac{dy}{dt} = 2e^t + 5e^{-t}$$

for all real values of t. At time $t = 0$, the particle is at the point $(4, 1)$.

(a) Find the speed of the particle and its acceleration vector at time $t = 0$.

(b) Find an equation of the line tangent to the path of the particle at time $t = 0$.

(c) Find the total distance traveled by the particle over the time interval $0 \le t \le 3$.

(d) Find the x-coordinate of the position of the particle at time $t = 3$.

(a) At time $t = 0$:

Speed $= \sqrt{x'(0)^2 + y'(0)^2} = \sqrt{3^2 + 7^2} = \sqrt{58}$

Acceleration vector $= \langle x''(0), y''(0) \rangle = \langle 0, -3 \rangle$

$2: \begin{cases} 1: \text{speed} \\ 1: \text{acceleration vector} \end{cases}$

(b) $\dfrac{dy}{dx} = \dfrac{y'(0)}{x'(0)} = \dfrac{7}{3}$

Tangent line is $y = \frac{7}{3}(x - 4) + 1$

$2: \begin{cases} 1: \text{slope} \\ 1: \text{tangent line} \end{cases}$

(c) Distance $= \displaystyle\int_0^3 \sqrt{\left(\sqrt{t^4 + 9}\right)^2 + \left(2e^t + 5e^{-t}\right)^2}\, dt$

$= 45.226$ or 45.227

$3: \begin{cases} 2: \text{distance integral} \\ \quad \langle -1 \rangle \text{ each integrand error} \\ \quad \langle -1 \rangle \text{ error in limits} \\ 1: \text{answer} \end{cases}$

(d) $x(3) = 4 + \displaystyle\int_0^3 \sqrt{t^4 + 9}\, dt$

$= 17.930$ or 17.931

$2: \begin{cases} 1: \text{integral} \\ 1: \text{answer} \end{cases}$