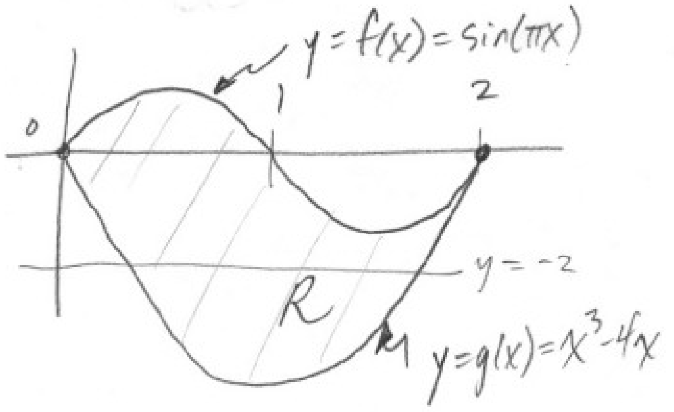
2008 BC Cal Exam

(1)



a) $A = \int_0^2 [f(x) - g(x)]\,dx$

$\approx \boxed{4}$

b) poi: $x^3 - 4x = -2$

$x = a = 0.539\ldots$

$x = b = 1.675\ldots$

$\boxed{A = \int_a^b (-2 - g(x))\,dx}$

c) $V = \int_0^2 (f(x) - g(x))^2\,dx$

$\approx \boxed{9.978}$

d) $V = \int_0^2 [(f(x) - g(x)) \cdot h(x)]\,dx$

$\approx \boxed{8.369 \text{ or } 8.370}$

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(2) a) $L'(5.5) \approx \frac{L(7) - L(4)}{7 - 4} = \frac{150 - 126}{3} = \boxed{8 \text{ people/hr}}$

b) Avg value $= \frac{\int_0^4 L(t)\,dt}{4} = \frac{\frac{1}{2}(120+156)(1) + \frac{1}{2}(156+176)(2) + \frac{1}{2}(176+126)(1)}{4}$

$\approx \frac{621}{4} = \boxed{155.25 \text{ people}}$

c) Since L' and L are differentiable and $L(t)$ changes from inc to dec or dec to inc at least 3 times, $L'(t) = 0$ $\boxed{\text{at least 3 times}}$ on $0 \le t \le 9$.

d) Tickets $= \int_0^3 r(t)\,dt \approx 972.784$ or about $\boxed{973 \text{ tickets}}$

-Koyuki

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(3) $\boxed{h(x) \approx T_1(x) = 80 + 128(x-2)}$

a) $h(1.9) \approx T_1(1.9) = 80 + 128(1.9-2) = \boxed{67.2}$

Since $h'(x)$ is increasing, $h''(x)$ is positive for all x in the interval, thus $\boxed{T_1(1.9) < h(1.9)}$

b) $\boxed{h(x) \approx T_3(x) = 80 + 128(x-2) + \frac{488}{3}\left(\frac{1}{2!}\right)(x-2)^2 + \frac{448}{3}\left(\frac{1}{3!}\right)(x-2)^3}$

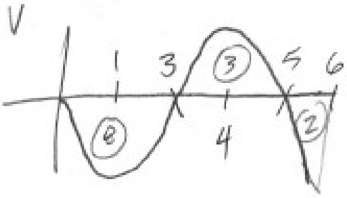
$h(1.9) \approx T_3(1.9) = 80 + 128(-.1) + \frac{244}{3}(-.1)^2 + \frac{448}{18}(-.1)^3$

$= \boxed{68.002519...}$

c) $|R_3(x)| = \left|\frac{f^4(z)}{4!}(x-2)^4\right|$

$\boxed{|R_3(1.9)| \leq \left|\left(\frac{584}{9}\right)\left(\frac{1}{4!}\right)(-.1)^4\right| = 2.7037... \times 10^{-4} < 3.000 \times 10^{-4}}$

(4)



a) Particle is furthest to left at the minimum position, $s(t)$. This occurs at endpts or critical values of $s(t)$.

$\boxed{\text{* this occurs at } t=3 \text{ with a value of } -10}$

$s(0) = -2$
$s(6) = -9$
$s(3) = -10$
$s(5) = -7$

(b) Since the integral function is continuous, the accumulated areas pass through -8 at least 3 times

(c) $\boxed{\text{on } 2<t<3\text{, since } v(t)<0 \text{ and } v'(t) = a(t) > 0\text{, speed is decreasing.}}$

(d) Acceleration is negative when $v'(t)$ (slopes of graph) is negative. this occurs on the open intervals $\boxed{(0,1) \cup (4,6)}$

-Korpi

2008 BC exam

(5) $f'(x) = (x-3)e^{x}$, $x>0$, $f(1)=7$.

(a) $f'(3)=0$

x	1	3	4
$f'(x)$	$-$		$+$

f has a Rel. Minimum @ $x=3$ since $f'(x)$ changes from neg to pos. there.

(b) f is decreasing when $f'<0$, this is when $x-3<0$ or $x<3$ & $x>0$ or $\boxed{0<x<3}$

f is cc up when $f''>0$

$$f''(x) = e^{x} + (x-3)e^{x} = e^{x}(x-2) = 0$$

$x=2$

x	1	2	3
$f''(x)$	$-$		$+$

$f''(x)$ cc up for $x>2$

So f is dec/cc up on $\boxed{2<x<3}$

(c) f(3):

$$f(3) = 7 + \int_1^3 (x-3)e^{x}\,dx$$

$u = x-3$, $dv = e^{x}$
$du = dx$, $v = e^{x}$

$$= 7 + \left[(x-3)e^{x}\Big|_1^3 - \int_1^3 e^{x}\,dx \right]$$

$$= 7 + \left[(x-3)e^{x} - e^{x} \right]\Big|_1^3 = 7 + \left[e^{x}(x-4) \right]\Big|_1^3$$

$$= 7 + \left[-e^{3} + 3e \right] = \boxed{3e - e^{3} + 7}$$

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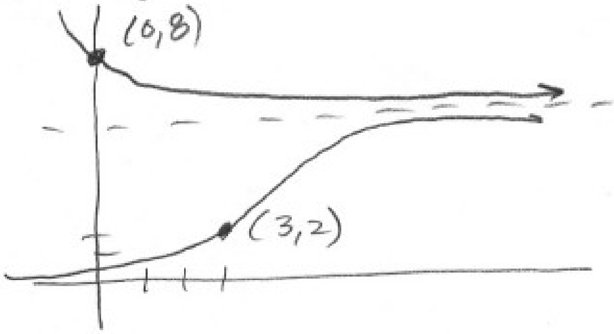
⑥ $\frac{dy}{dt} = \frac{y}{8}(6-y)$. $f(0)=8$ ← Logistic

ⓐ



ⓑ 2 steps: $\Delta t = \frac{1-0}{2} = 0.5$

t	y	m	Δy	y new
0	8	-2	-1	7
0.5	7	$-\frac{7}{8}$	$-\frac{7}{16}$	$\frac{105}{16}$
1	$\frac{105}{16}$			

$$\boxed{f(1) \approx \frac{105}{16} = 6.5625}$$

c) $f(0)=8$

$f'(0)=-2$

$f''(0) = \frac{3}{4}(-2) - \frac{1}{4}(8)(-2)$

$= -\frac{3}{2} + 4 = \frac{5}{2}$

Find $f''(t)$:

$y' = \frac{y}{8}(6-y)$

$y' = \frac{3}{4}y - \frac{1}{8}y^2$

$y'' = \frac{3}{4}y' - \frac{1}{4}yy'$

$$\boxed{f(t) \approx P_2(t) = 8 - 2x + \left(\frac{5}{2}\right)\left(\frac{1}{2!}\right)x^2}$$

$$f(1) \approx P_2(1) = 8 - 2 + \frac{5}{4} = \boxed{\frac{29}{4} = 7.25}$$

d) Range of f, $t \geq 0$:

since $f(0)=8$, the graph starts Above the carrying capicity, so the values will decrease monotonically toward $y=6$, that is $\lim_{t\to\infty} f(t) = 6$

So the Range is $\boxed{(6,8]}$ or $6 < f(t) \leq 8$

– Koji