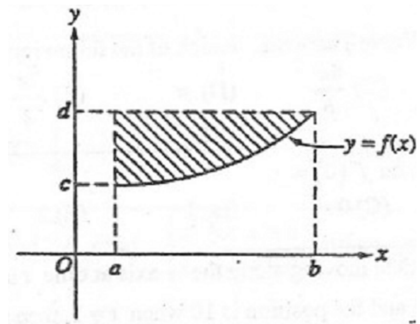
AB Review 01, **No** Calculator Permitted



1. Which of the following represents the area of the shaded region in the figure above?

(A) $\int_c^d f(y)\,dy$ (B) $\int_a^b (d - f(x))\,dx$ (C) $f'(b) - f'(a)$

(D) $(b-a)[f(b) - f(a)]$ (E) $(d-c)[f(b) - f(a)]$

2. If $x^3 + 3xy + 2y^3 = 17$, then in terms of x and y, $\frac{dy}{dx} =$

(A) $-\frac{x^2+y}{x+2y^2}$ (B) $-\frac{x^2+y}{x+y^2}$ (C) $-\frac{x^2+y}{x+2y}$ (D) $-\frac{x^2+y}{2y^2}$ (E) $-\frac{x^2}{1+2y^2}$

3. $\int \frac{3x^2}{\sqrt{x^3+1}}\,dx =$

(A) $2\sqrt{x^3+1} + C$ (B) $\frac{3}{2}\sqrt{x^3+1} + C$ (C) $\sqrt{x^3+1} + C$ (D) $\ln\sqrt{x^3+1} + C$ (E) $\ln(x^3+1) + C$

4. For what value of x does the function $f(x)=(x-2)(x-3)^2$ have a relative maximum?

(A) -3 (B) $-\frac{7}{3}$ (C) $-\frac{5}{2}$ (D) $\frac{7}{3}$ (E) $\frac{5}{2}$

5. If $f(x)=\sin\left(\frac{x}{2}\right)$, then there exists a number c in the interval $\frac{\pi}{2}<x<\frac{3\pi}{2}$ that satisfies the conclusion of the Mean Value Theorem. Which of the following could be c?

(A) $\frac{2\pi}{3}$ (B) $\frac{3\pi}{4}$ (C) $\frac{5\pi}{6}$ (D) π (E) $\frac{3\pi}{2}$

6. If $f(x)=(x-1)^2\sin x$, then $f'(0)=$

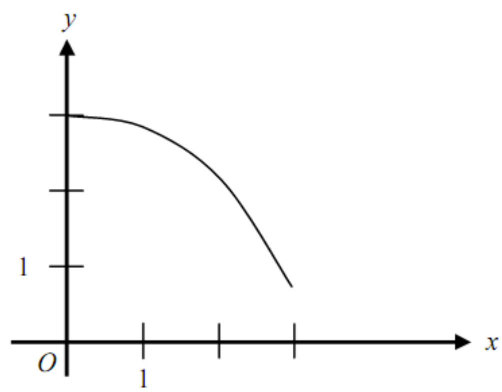
(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

7. The acceleration of a particle moving along the x-axis at time t is given by $a(t)=6t-2$. If the velocity is 25 when $t=3$ and the position is 10 when $t=1$, then the position $x(t)=$

(A) $9t^2+1$ (B) $3t^2-2t+4$ (C) t^3-t^2+4t+6 (D) $t^3-t^2+9t-20$ (E) $36t^3-4t^2-77t+55$

8. $\dfrac{d}{dx}\displaystyle\int_0^x \cos(2\pi u)\,du$ is

(A) 0 (B) $\dfrac{1}{2\pi}\sin x$ (C) $\dfrac{1}{2\pi}\cos(2\pi x)$ (D) $\cos(2\pi x)$ (E) $2\pi\cos(2\pi x)$



Graph of f

9. The graph of the function f is shown above for $0 \le x \le 3$. Of the following, which has the least value?

(A) $\displaystyle\int_1^3 f(x)\,dx$

(B) Left Riemann sum approximation of $\displaystyle\int_1^3 f(x)\,dx$ with 4 subintervals of equal length.

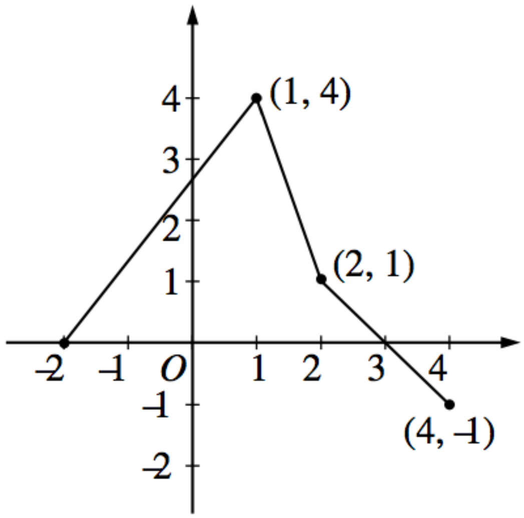
(C) Right Riemann sum approximation of $\displaystyle\int_1^3 f(x)\,dx$ with 4 subintervals of equal length.

(D) Midpoint Riemann sum approximation of $\displaystyle\int_1^3 f(x)\,dx$ with 4 subintervals of equal length.

(E) Trapezoidal sum approximation of $\displaystyle\int_1^3 f(x)\,dx$ with 4 subintervals of equal length.

10. What is the minimum value of $f(x) = x\ln x$?

(A) $-e$ (B) -1 (C) $-\dfrac{1}{e}$ (D) 0 (E) $f(x)$ has no minimum value.



11. (1999, AB-5) The graph of the function *f*, consisting of three line segments, is shown above. Let $g(x) = \int_1^x f(t)\,dt$.

(a) Compute $g(4)$ and $g(-2)$.

(b) Find the instantaneous rate of change of *g*, with respect to *x*, at $x = 1$.

(c) Find the absolute minimum value of *g* on the closed interval $[-2, 4]$. Justify your answer.

(d) The second derivative of *g* is not defined at $x = 1$ and $x = 2$. How many of these values are *x*-coordinates of points of inflection of the graph of *g*? Justify your answer.

12. (1998, AB-4) Let f be a function with $f(1)=4$ such that for all points (x, y) on the graph of f the slope is given by $\frac{3x^2+1}{2y}$.

(a) Find the slope of the graph of f at the point where $x=1$.

(b) Write an equation for the line tangent to the graph of f at $x=1$, and use it to approximated $f(1.2)$.

(c) Find $f(x)$ by solving the separable differential equation $\frac{dy}{dx}=\frac{3x^2+1}{2y}$ with the initial condition $f(1)=4$.

(d) Use your solution from part (c) to find $f(1.2)$.