AB Review 14 No Calculator permitted (except on #11)

1.

If $y = x^2 \sin(2x)$, then $\frac{dy}{dx} =$

A) $2x\cos(2x)$

B) $4x\cos(2x)$

C) $2x[\sin(2x) + \cos(2x)]$

D) $2x[\sin(2x) - x\cos(2x)]$

E) $2x[\sin(2x) + x\cos(2x)]$

2.

Let f be the function given by $f(x) = 2xe^x$. The graph of f is concave down when

A) $x < -2$ B) $x > -2$ C) $x < -1$ D) $x > -1$ E) $x < 0$

3.

A curve has a slope $2x + 3$ at each point (x,y) on the curve. Which of the following is an equation for this curve if it passes through the point $(1,2)$?

A) $y = 5x - 3$

B) $y = x^2 + 1$

C) $y = x^2 + 3x$

D) $y = x^2 + 3x - 2$

E) $y = 2x^2 + 3x - 3$

4.

x	-4	-3	-2	-1	0	1	2	3	4
$g'(x)$	2	3	0	-3	-2	-1	0	3	2

The derivative g' of a function g is continuous and has exactly two zeros. Selected values of g' are given in the table above. If the domain of g is the set of all real numbers, then g is decreasing on which of the following intervals?

A) $-2 \le x \le 2$ only

B) $-1 \le x \le 1$ only

C) $x \ge -2$

D) $x \ge 2$ only

E) $x \le -2$ or $x \ge 2$

5.

$$f(x) = \begin{cases} x+2 & \text{if } x \le 3 \\ 4x-7 & \text{if } x > 3 \end{cases}$$

Let f be the function given above. Which of the following statements are true about f?

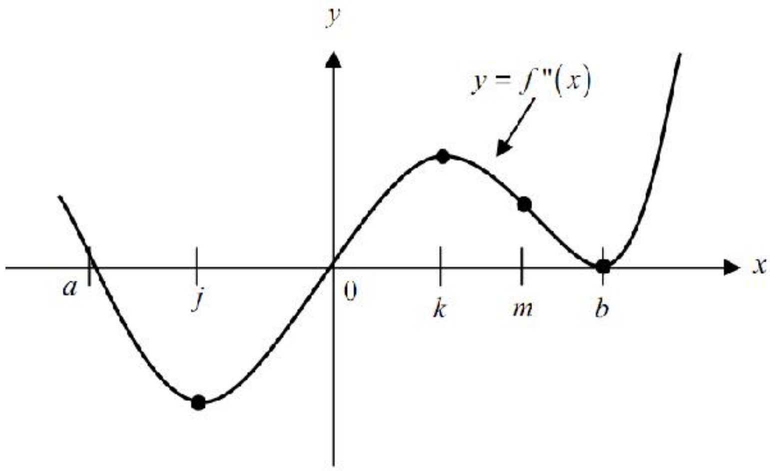
I. $\lim_{x \to 3} f(x)$ exists

II. f is continuous at x=3

III. f is differentiable at x=3

(A) None (B) I only (C) II only (D) I and II only (E) I, II, and III

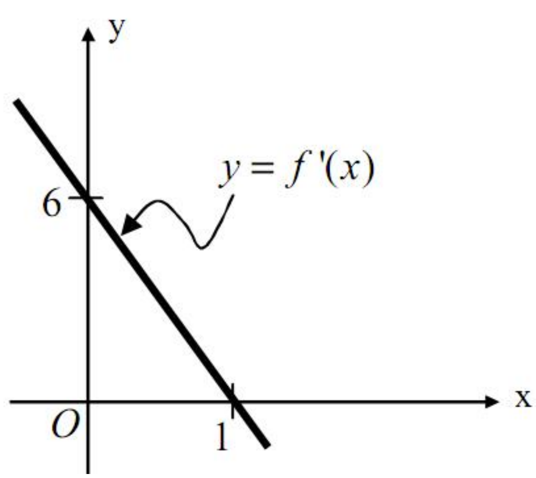
6.



The second derivative of the function f is given by $f''(x) = x(x-a)(x-b)^2$. The graph of f'' is shown above. For what values of x does the graph of f have a point of inflection?

A) 0 and a only B) 0 and m only C) b and j only D) 0, a, and b E) b, j, and k

7.



The graph of f', the derivative of f, is the line shown in the figure above. If $f(0) = 5$, then $f(1) =$

A) 0 B) 3 C) 6 D) 8 E) 11

8.

$$\frac{d}{dx}\left(\int_0^{x^2} \sin(t^3)dt\right) =$$

A) $-\cos(x^6)$ B) $\sin(x^3)$ C) $\sin(x^6)$ D) $2x\sin(x^3)$ E) $2x\sin(x^6)$

9.

What is the slope of the line tangent to the curve $3y^2 - 2x^2 = 6 - 2xy$ at the point (3,2)?

A)0 B) $\frac{4}{9}$ C) $\frac{7}{9}$ D) $\frac{6}{7}$ E) $\frac{5}{3}$

10.

Let f be the function defined by $f(x) = x^3 + x$. If $g(x) = f^{-1}(x)$ and g(2)=1, what is the value of $g'(2)$?

A) $\frac{1}{13}$ B) $\frac{1}{4}$ C) $\frac{7}{4}$ D) 4 E) 13

11. (2011, AB-2) (**Calculator Permitted**)

t (minutes)	0	2	5	9	10
$H(t)$ (degrees Celsius)	66	60	52	44	43

As a pot of tea cools, the temperature of the tea is modeled by a differentiable function H for $0 \le t \le 10$, where time t is measured in minutes and temperature $H(t)$ is measured in degrees Celsius. Values of $H(t)$ at selected values of time t are shown in the table above.

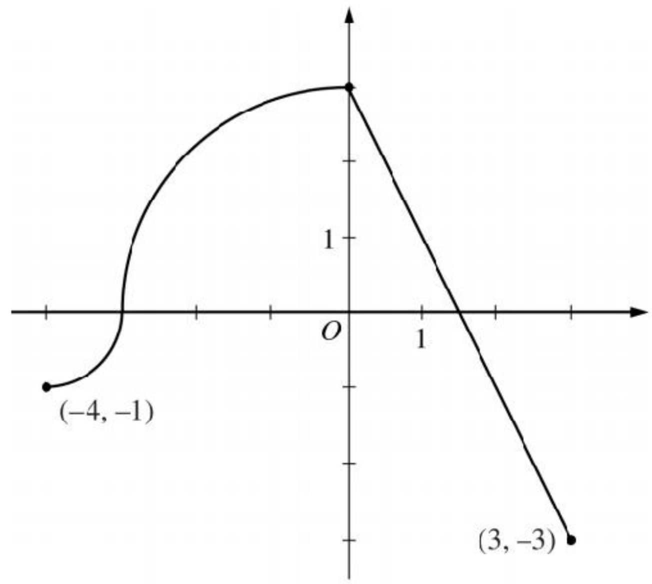
(a) Use the data in the table to approximate the rate at which the temperature of the tea is changing at time $t = 3.5$. Show the computations that lead to your answer.

(b) Using correct units, explain the meaning of $\frac{1}{10}\int_0^{10} H(t)\,dt$ in the context of this problem. Use a trapezoidal sum with the four subintervals indicated by the table to estimate $\frac{1}{10}\int_0^{10} H(t)\,dt$.

(c) Evaluate $\int_0^{10} H'(t)\,dt$. Using correct units, explain the meaning of the expression in the context of this problem.

(d) At time $t = 0$, biscuits with temperature 100°C were removed from an oven. The temperature of the biscuits at time t is modeled by a differentiable function B for which it is known that $B'(t) = -13.84e^{-0.173t}$. Using the given models, at time $t = 10$, how much cooler are the biscuits than the tea?

12. (2011, AB-4)



Graph of f

The continuous function f is defined on the interval $-4 \le x \le 3$. The graph of f consists of two quarter circles and one line segment, as shown in the figure above. Let $g(x) = 2x + \int_0^x f(t)\,dt$.

(a) Find $g(-3)$. Find $g'(x)$ and evaluate $g'(-3)$.

(b) Determine the x-coordinate of the point at which g has an absolute maximum on the interval $-4 \le x \le 3$. Justify your answer.

(c) Find all values of x on the interval $-4 < x < 3$ for which the graph of g has a point of inflection. Give a reason for your answer.

(d) Find the average rate of change of f on the interval $-4 \le x \le 3$. There is no point c, $-4 < c < 3$, for which $f'(c)$ is equal to that average rate of change. Explain why this statement does not contradict the Mean Value Theorem.