Download at Bottom of Calculus Maximus

Korpi's Last-Minute AP Things Not to Forget to Remember

use a pencil

#15 2017

- Integration by u-substitution
    - When your "inside" function is linear, and it's derivative is "off" by more than a constant. ($y = mx + b$)
    - Let $u =$ "inside" function and change all variables from x, dx to u, du.
    - **Example**: $\displaystyle\int_0^1 \frac{2x}{\sqrt{x+2}}\,dx$ ⇒ "off by x"

$u = x + 2$

$\frac{du}{dx} = 1$

$du = dx$

$x = u - 2$

Solve for dx

$\displaystyle\int_2^3 \frac{2(u-2)}{\sqrt{u}}\,du$

$\displaystyle\int_2^3 (2u-4)u^{-1/2}\,du$

$\displaystyle\int_2^3 \left(2u^{1/2} - 4u^{-1/2}\right)du$

$(2)\left(\frac{2}{3}\right)u^{3/2} - (4)(2)u^{1/2}\Big|_2^3$

$\left[\frac{4}{3}(3)^{3/2} - 8(3)^{1/2}\right] - \left[\frac{4}{3}(2^{3/2}) - 8(2^{1/2})\right]$

or $\displaystyle\int_1^4 x\sqrt{3x^2-1}\,dx$

$u = 3x^2 - 1$

$\frac{du}{dx} = 6x$

$du = 6x\,dx$

$x\,dx = \frac{1}{6}du$

$\displaystyle\frac{1}{6}\int_2^{47} \sqrt{u}\,du$

- Logarithmic Differentiation (LOG DIFF)
    - When you are taking the derivative of a variable function raised to a power of a variable function.
    - Take the natural log of both sides, differentiate implicitly, then resolve for y.
    - **Example**: $\frac{d}{dx}\left[x^{\sin x}\right] =$ $(\text{var})^{\text{var}}$

$y = x^{\sin x}$

$\ln y = \sin x \cdot \ln x$

$\frac{d}{dx}: \frac{1}{y}\cdot\frac{dy}{dx} = \cos x \cdot \ln x + \sin x\left(\frac{1}{x}\right)$

$\frac{dy}{dx} = \left[\cos x \cdot \ln x + \sin x\left(\frac{1}{x}\right)\right]\cdot x^{\sin x}$

$y = \frac{a^n b^m}{c^p d^r}$

$\ln y = n\ln a + m\ln b - p\ln c - r\ln d$

$\frac{d}{dx}:$

- Derivatives of Trig/inverse Trig functions
    - $\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}}$ $\quad\frac{d}{dx}\arctan x = \frac{1}{1+x^2}$ $\quad\frac{d}{dx}\operatorname{arcsec} x = \frac{1}{|x|\sqrt{x^2-1}}$
    - $\frac{d}{dx}\arccos x = \frac{-1}{\sqrt{1-x^2}}$ $\quad\frac{d}{dx}\operatorname{arccot} x = \frac{-1}{1+x^2}$ $\quad\frac{d}{dx}\operatorname{arccsc} x = \frac{-1}{|x|\sqrt{x^2-1}}$
    - $\frac{d}{dx}\sin x = \cos x$ $\quad\frac{d}{dx}\tan x = \sec^2 x$ $\quad\frac{d}{dx}\sec x = \sec x\tan x$
    - $\frac{d}{dx}\cos x = -\sin x$ $\quad\frac{d}{dx}\cot x = -\csc^2 x$ $\quad\frac{d}{dx}\csc x = -\csc x\cot x$

$\sin^2 x = (\sin x)^2$

- Example: $\frac{d}{dx}\cos^{-1}\left(\tan\left(2x^2\right)\right) =$

$\frac{-1}{\sqrt{1-\left(\tan(2x^2)\right)^2}}\cdot\sec^2(2x^2)\cdot 4x$

- Integral of Trig functions
  - $\int \sin x\,dx =$ $-\cos x + C$
    $\int \cos x\,dx =$ $\sin x + C$
    $\int \tan x\,dx =$ $\int \frac{\sin x}{\cos x}dx$; $-\ln|\cos x| + C$; $\ln|\sec x| + C$
    $\int \cot x\,dx =$ $\ln|\sin x| + C$; $\frac{1}{\sin x}(\cos x)$; ~~$-\ln|\csc x| + C$~~
  - $\int \sec x\,dx =$ $\ln|\sec x + \tan x| + C$
    $\int \csc x\,dx =$ $-\ln|\csc x + \cot x| + C$
    $\int \sec^2 x\,dx =$ $\tan x + C$
    $\int \sec x \tan x\,dx =$ $\sec x + C$
  - **Examples**: $\int \tan^2 x\,dx =$ $\int(\sec^2 x - 1)dx =$ $\boxed{\tan x - x + C}$

    PIDs: $\cos^2 x + \sin^2 x = 1$
    $1 + \tan^2 x = \sec^2 x$
    $1 + \cot^2 x = \csc^2 x$

    $\int \frac{\cot(\sqrt{x})}{\sqrt{x}}dx =$ $2\ln|\sin\sqrt{x}| + C$

    $x^{1/2} \xrightarrow{\text{deriv}} \frac{1}{2}x^{-1/2} = \frac{1}{2}\left(\frac{1}{\sqrt{x}}\right)$

    $\int \cot(x^{1/2}) \cdot x^{-1/2}\,dx$; $\frac{1}{2}x^{-1/2}$

- Finding extrema vs. finding the location of extrema
  - An extreme value is a y-value. It occurs at an x-value
  - **Example**: Find the maximum value of $f(x) = x^2 - 5$ on $[-1,2]$ EVT

    $f'(x) = 2x = 0$
    $x = 0 \in [-1,2]$
    $f(-1) = -4$
    $f(2) = -1$
    crit $f(0) = -5$
    So f has a max of -1

- Finding slopes of inverse functions
  - Inverse functions, at corresponding points, have reciprocal slopes.
  - If $f(g(x)) = x = g(f(x))$, then $f(x)$ and $g(x)$ are inverses
  - $g(a) = b$ implies $f(b) = a$
    g: (a, b)
    f: (b, a)
  - $g'(a) = \frac{1}{f'(b)}$ $g'(a) = \frac{1}{f'(b)}$
    $f(-1) = -2$
    $f(-2) = 7$
  - **Example**: If $f(x) = g^{-1}(x)$ and $f(x) = 2x^2 + 3x - 1$ and if $g(-2) = -1$, find $g'(-2)$.

    $f'(x) = 4x + 3$
    $f'(-1) = -4 + 3$
    $f'(-1) = \boxed{-1}$

    g: (−2, −1)
    f: (−1, −2)
    output

    So $g'(-2) = \frac{1}{f'(-1)}$
    $= \frac{1}{-1}$
    $= \boxed{-1}$

- Finding the slope of a normal line to a function, $f(x)$, at a point $x = a$
  - Normal lines are perpendicular to tangent lines at a point.
  - The normal slope, n, is the opposite, reciprocal of the tangent slope.
  - $n = \dfrac{-1}{f'(a)}$
  - **Example**: Find the equation of the normal line to the graph of $y = e^{2x}$ at $x = \ln 2$

$y' = e^{2x} \cdot 2$ $\quad (x, y) = (\ln 2, e^{2(\ln 2)})$

$y'(\ln 2) = 4 \cdot 2 = 8 = m$ $\quad (\ln 2, e^{\ln 2^2})$

$\boxed{n = -\frac{1}{8}}$ $\quad \boxed{(\ln 2, 4)}$

eq: $y = 4 - \frac{1}{8}(x - \ln 2)$

- Squiggle Alert when the words "approximate" or "estimate" are used in the question
  - Explicitly stated approximations must have an approximation symbol, $\approx$, rather than an equal sign.
  - This happens with tangent line approximations, numeric methods of integration, linearization, Euler's Method (BC), and Taylor Polynomials (BC).
  - **Example**: If f is differentiable, and if $f(3) = 2$ and $f(5.5) = 5$, approximate $f'(4.1)$. Show the work that leads to your answer.

vertangent cusp alert $D_{f'}$

$\boxed{f(x) = x^{2/3}}$ $\leftarrow 0 < a < 1$

ex) $f(x) = 3x + (x+7)^{3/5}$

$D_f: \mathbb{R}$ $\quad D_{f'}: x \neq -7$

$f'(4.1) \approx \boxed{\dfrac{5-2}{5.5-3}}$ DQ

$\approx \boxed{\dfrac{f(5.5) - f(3)}{5-3}}$

Indicate your method

- Average value vs. average rate of change
  - Average value is the average y-value for whatever is being measured on the y-axis.
  - Average value is "integral over width"
  - Average value $= \dfrac{\int_a^b f(x)\,dx}{b-a} = \dfrac{1}{b-a}\int_a^b f(x)\,dx$

use their words

  - Average rate of change is the change in y over the change in x—the slope of the secant line
  - Average rate of change $= \boxed{\dfrac{f(b) - f(a)}{b-a}}$ DQ
  - **Example**: If $W(t)$ is the rate at which rain falls on a roof of a house for $0 \le t \le 3$, where W is measured in $\boxed{\text{cm/hr}}$ and t is measured in hours. Explain the meaning, with correct units, of $\dfrac{1}{3}\int_0^3 W(t)\,dt$ in the context of rainfall.

cm/hr

$\boxed{\text{Avg}}$ rate at which rain falls on roof ~~...~~ in cm/hr $\leftarrow$ dep

~~on roof~~ from $t = 0$ hr to $t = 3$ hr. $\leftarrow$ indep

$\int_4^7 \sin x\,dx = 3.461....$

- The misuse of equality a.k.a. mathematically prevarication = lie
  - If you use an equal sign, the expressions you are equating better be equal, or you will lose a point.
  - **Example**: If $f(x)=2x^2-1$, find $f'(1)$
    - WRONG: $f'(x)=4x=4$
    - CORRECT: $f'(x)=4x$, $f'(1)=4(1)=4$

therefore

$\Rightarrow$

Loaded term

So, . . .

  - **Example**: To the nearest whole number, find $\int_0^2 e^x\,dx$
    - WRONG: $\int_0^2 e^x\,dx = 6.389 = 6$
    - CORRECT: $\int_0^2 e^x\,dx = 6.389 \approx 6$ OR $\int_0^2 e^x\,dx = 6$

- Be sure to include units in all final numeric answers AND any written explanation of this answer (including both independent AND dependent variables.
  - Units can cost you an entire point if you omit them or use the wrong ones.
  - **Example**: $w(t)$ is the temperature of water in a jug in a refrigerator, in $^\circ F$, where t is in minutes.

    In the context of the problem, explain the meaning of (a) $w'(5)=-2.1$ (b) $\frac{1}{5}\int_0^5 w(t)\,dt = 44$

7 ft/sec ✓ but ~~ft/sec~~

At t=5min, the water's temp is decreasing by 2.1 °F per min.

from t=0min to t=5min, the avg temp of water is 44°F

- Speed increasing or decreasing vs. Velocity increasing or decreasing
  - If at $t=c$, $v(c)>0$ AND $a(c)>0$ or $v(c)<0$ AND $a(c)<0$, then speed is increasing at $t=c$.
  - If at $t=c$, $v(c)<0$ AND $a(c)>0$ or $v(c)>0$ AND $a(c)<0$, then speed is decreasing at $t=c$.
  - If at $t=c$, $v'(c)=a(c)>0$, then velocity is increasing at $t=c$.
  - If at $t=c$, $v'(c)=a(c)<0$, then velocity is decreasing at $t=c$.
  - If the graph of $v(t)$ moves TOWARD the t axis, speed is decreasing.
  - If the graph of $v(t)$ moves AWAY FROM the t axis, speed is increasing.
  - **Example**: If a particle moves along the x-axis such that for $t\ge 0$, its position is give by , $x(t)=\frac{1}{3}t^3-4t^2+15t-7$, at $t=4.5$, is the speed of the particle increasing or decreasing? At this time is the velocity of the particle increasing or decreasing? Justify your answers.

$x'(t)=v(t)=t^2-8t+15$, $v(4.5)=-0.75<0$

$v'(t)=a(t)=2t-8$, $\boxed{a(4.5)=1>0}$

So at $t=4.5$ sec, the speed is decreasing.

what the signs? + or −

How a sign changes

But, since $v'(4.5)=a(4.5)=1>0$, at $t=4.5$, velocity is increasing.

- IVT (The Intermediate Value Theorem)
  - If $f(x)$ is continuous on a closed interval $[a,b]$, then $f(x)$ takes on all y-values between $f(a)$ and $f(b)$.
  - **Example**: If $f(x)$ is a differentiable function such that $f(-1)=-3$ and $f(4)=\frac{5}{6}$, explain why $f(x)$ must have a root on the interval $(-1,4)$.

$f(x)=0$

$f(x)$ is differentiable, so $f(x)$ is continuous on $[-1,4]$. Since f has a root when $f(c)=0$ for some $x=c$, and since $f(-1)=-3<0<\frac{5}{6}=f(4)$, by the IVT, f must have a ~~zero~~ root on $(-1,4)$.

use their words

Prove a y-value exists, use IVT (yVT)

- EVT (The Extreme Value Theorem)
  - If $f(x)$ is continuous on a closed interval $[a,b]$, then $f(x)$ has both a Maximum and Minimum on the CLOSED interval $[a,b]$.
  - **Example**: If a particle moves along the x-axis such that its position is give by, $x(t)=t^2-3t+2$, on the interval $0 \le t \le 2$, at what time, t, is the particle farthest left? Farthest right?

$x(t)$ is continuous $\forall x \in \mathbb{R}$

minimum position Max position

$x'(t)=2t-3=0$

$\boxed{t=\frac{3}{2}}$ critical value

endpts $\begin{cases} x(0)=2 \leftarrow \text{MAX} \\ x(2)=0 \end{cases}$

c.v. $\{ x(\frac{3}{2})=\frac{9}{4}-\frac{9}{2}+2=-\frac{1}{4} \leftarrow \text{MIN}$

So, particle is farthest left at $t=3/2$ & farthest right at $t=0$.

Prove a slope $= m$ (MVT)

- MVT (The Mean Value Theorem)
  - If $f(x)$ is continuous on a closed interval $[a,b]$, and differentiable on the open interval (a,b), then there is an $x=c$ on the OPEN interval (a,b), where the slope of the tangent line (instantaneous rate of change/derivative) equals the slope of the secant line (average rate of change).
  - USED TO SHOW THAT A DERIVATIVE EXISTS ON AN INTERVAL
  - Set it up as $\boxed{f'(x)=\dfrac{f(b)-f(a)}{b-a}}$, then solve for x, then make sure x is in the OPEN interval!
  - **Example**: If $f(x)=x^3+x-4$, on the interval $-1 \le x \le 2$, find the value of c guaranteed by the Mean Value Theorem.

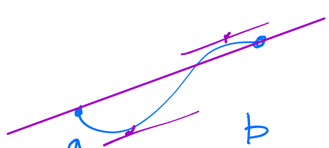
$f'(x)=\dfrac{f(2)-f(-1)}{2-(-1)}$

$3x^2+1=\dfrac{6+6}{3}=4$

$\begin{matrix} 3x^2+1=4 \\ 3x^2=3 \end{matrix} \Big\{ \boxed{x^2=1}$ $\boxed{x=-1}$, $\boxed{x=1}$

interior only

only since $x=-1 \notin (-1,2)$ open interval



  - **Example:** If $f'(x)$ is a differentiable function for all x, and if $f'(5)=-2$ and $f'(7)=4$, explain why there must be a c, $5<c<7$ such that $f''(c)=3$.

Since f' is differentiable, it is continuous by the MVT: $f''(c)=\boxed{\dfrac{f'(7)-f'(5)}{7-5}}=\dfrac{4+2}{2}=3$ for some $5<c<7$.

MVT

if $f(3)=4$ & $f'\le 3$, what is the max value of $f(5)$?

$\frac{f(5)-f(3)}{5-3} = f'(x) \le 3$, $f(5) - 4 \le 6$, $f(5) \le 10$

- Geometric formulas to remember
  - Volume of a Sphere: $V = \frac{4}{3}\pi r^3$ — Surface area of a Sphere: $A = 4\pi r^2$
  - Volume of a Cone: $V = \frac{\pi}{3} r^2 h$ — Volume of a Cylinder: $V = \pi r^2 h$
  - Surface area of a Cylinder: $A = 2\pi r^2 + 2\pi r h$ — Equilateral Triangle: $A = \frac{\sqrt{3}}{4} s^2$
  - Trapezoid: $A = \frac{1}{2}\Delta x\left(y_1 + y_2\right)$ — Rectangle: $A = h \cdot w$

- Justifying relative extrema using the First Derivative test and Second Derivative Test
  - First Derivative Test (at a critical point, $(c, f(c))$)
    - "Since $f'(c) = 0$ (or $f'(c) = DNE$), and since $f'(x)$ changes from positive to negative at $x = c$, $f(x)$ has a Relative (local) Maximum at $x = c$."
    - "Since $f'(c) = 0$ (or $f'(c) = DNE$), and since $f'(x)$ changes from negative to positive at $x = c$, $f(x)$ has a Relative (local) Minimum at $x = c$."
  - Second Derivative Test (at a critical value $(c, f(c))$)
    - "Since $f'(c) = 0$ (or $f'(c) = DNE$), and since $f''(c) < 0$, $f(x)$ has a Relative (local) Maximum at $x = c$."
    - "Since $f'(c) = 0$ (or $f'(c) = DNE$), and since $f''(c) > 0$, $f(x)$ has a Relative (local) Minimum at $x = c$."

$x = c$

Squeeze

$\lim_{x\to c} f(x) = ?$

if $g(x) \le f(x) \le h(x)$ $\forall x \ne c$

then if $\lim_{x\to c} g(x) = L = \lim_{x\to c} h(x)$,

then $\lim_{x\to c} f(x) = L$ also!

- Justifying an inflection point at a p.i.v. (possible inflection value)
  - If $f(c)$ is defined, and either $f''(c) = 0$ or $f''(c) = DNE$, then $f(x)$ has an inflection point at $(c, f(c))$ if f'' changes from positive to negative at $x = c$ or negative to positive at $x = c$.
  - **Example**: A continuous function $f(x)$ has a second derivative $f''(x) = \frac{|x-3|}{x-3}$. Determine if $f(x)$ has an inflection value or not at $x = 3$. Justify.

$\frac{0}{0} \to$ Jump

Jump Man

ex1) $\lim_{x\to 3^-} \frac{|x-3|}{x-3} = -1$

ex2) $\lim_{x\to 3^+} \frac{x^2(x-3)}{|x-3|} = \lim_{x\to 3^+} x^2 \cdot \lim_{x\to 3^+} \frac{(x-3)}{|x-3|}$ = 9 · 1

4, 5, 6

$\lim_{x\to 3^+} \frac{2x-6}{|4x-12|}$ $\frac{0}{0}$

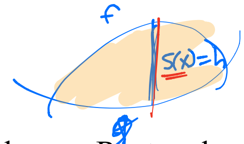
$f''(x) = DNE$ @ $x = 3$

x	2	3	4
f''	$-$		$+$

Since f'' changes from positive to negative at $x=3$, f has an inflection value @ $x=3$

$S(x)$

$Vol = M \cdot \int_a^b (s(x))^2\,dx$ T − B R − L



$s(x) = f - g$

- Cross-sectional volume magic numbers
  - Squares: 1 Equilateral Triangles: $\frac{\sqrt{3}}{4}$ Semicircles: $\frac{\pi}{8}$ Rectangles with height n times the base n
  - Quarter Circles: $\frac{\pi}{4}$ Isos Rt Triangle, Leg in Base: $\frac{1}{2}$ Isos Rt Triangle, Hypot in Base: $\frac{1}{4}$

- Inverse Trig Integral formulas
  - $\int \frac{du}{a^2+u^2} =$ $\frac{1}{a}\arctan\frac{u}{a} + C$
    $\int \frac{du}{\sqrt{a^2-u^2}} =$ $\arcsin\frac{u}{a} + C$ No $\frac{1}{a}$ in front
    $\int \frac{du}{u\sqrt{u^2-a^2}} =$ $\frac{1}{a}\,\text{arcsec}\frac{|u|}{a} + C$ create what you need (legally)
  - **Examples**:
    $\int \frac{1}{x^2+5}dx =$ $u = x$ $a = \sqrt{5}$ $\frac{1}{\sqrt{5}}\arctan\frac{x}{\sqrt{5}} + C$
    $\int \frac{5}{\sqrt{7-9x^2}}dx =$ $a = \sqrt{7}$ $u = 3x$ $(5)\left(\frac{1}{3}\right)\arcsin\frac{3x}{\sqrt{7}} + C$ rider corr
    $\int \frac{1e^x}{e^x\sqrt{e^{2x}-2}}dx =$ $u = e^x$ $a = \sqrt{2}$ $\left(e^{2x}\right)^{1/2} = \left(u^2\right)^{1/2}$ $u = e^x$
    $\frac{1}{\sqrt{2}}\,\text{arcsec}\frac{|e^x|}{\sqrt{2}} + C$
    $\frac{1}{\sqrt{2}}\,\text{arcsec}\frac{e^x}{\sqrt{2}} + C$

Parting C

- BC Convergence Tests (used also to determine endpoints of intervals of convergence)
  - *n*th term test for divergence
    $\sum a_n$ if $\lim_{n\to\infty} a_n \neq 0$, series diverges
  - Geometric series
    $\sum a \cdot r^n$, converges to $\frac{1\text{st term}}{1-r}$ if $|r|<1$ otherwise, diverges
    $\sum \frac{(-2)^n}{e^{n+1}} = \sum \frac{(-2)^n}{e^n e} = \sum \frac{1}{e}\cdot\left(\frac{-2}{e}\right)^n$
  - *p*-series
    $\sum \frac{1}{n^p}$ converges if $p>1$ Diverges if $p \le 1$
    $* \int_1^\infty \frac{1}{x^p}dx$, converge if $p>1$ to $\frac{1}{p-1}$
  - Direct/Limit Comparison Test
    Compare to a known series
  - Integral Test
    if $\int_1^\infty f(x)dx$ converges or Diverges, $\sum_{n=1}^\infty a_n$ converges too or Diverges
  - Ratio Test $\sum a_n$ converges if $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| < 1$
    diverges if > 1
    * to find Interval/Rad of Convergence
  - Alternating Series Test/Error
    $\sum (-1)^n a_n$
    converges if $\lim_{n\to\infty} a_n = 0$
    $(-1)^n, (-1)^{n+1}, (-1)^{n-1}, \cos(\pi n)$, $\cancel{(-1)^{2n}}$, $\cancel{(-1)^{2n+1}}$
- Pen or Pencil on the exam???
  Black or Dark Blue/Blue (No Lavender w/sparkles) ~~erase~~
- 3-decimal accuracy (round or truncate). Store non-exact answers needed for future calculations and LABEL THEM ON YOUR PAPER. Avoid duplicate letters. NEVER use an approximate answer to calculate a subsequent value.

- Implicit Differentiation—your derivative will have both x and y in it.
  - You will have as many $\frac{dy}{dx}$'s in your derivative as you have y's in your equation.
  - Look to solve for y first, especially if your answer choices are in terms of x only and/or you are finding an actual value and are only given $x = a$.
  - When solving for $\frac{dy}{dx}$, if you ever end up with an answer like $\frac{dy}{dx} = \frac{a-b}{c-d}$, realize that this is equivalent to $\frac{dy}{dx} = \frac{b-a}{d-c}$.
  - When finding a second derivative (or higher order derivative) implicitly, if the instructions say "in terms of x and y," be sure to plug in your $\frac{dy}{dx}$ expression into your final answer.

- Parenthesis are your best friends—it's better to have them and not need them than to need them and not have them. This is especially true for:
  - $k\int_a^b f(x)\,dx = k\left[\left(f(b)\right)-\left(f(a)\right)\right]$ $\qquad$ $\int_a^b \left[f(x)+g(x)\right]dx$
  - $\pi\int_a^b \left[\left(R(x)\right)^2 - \left(r(x)\right)^2\right]dx$ $\qquad$ $\lim_{h\to 0}\frac{\left((x+h)^2-2(x+h)+1\right)-\left(x^2-2x+1\right)}{h} =$
  - $5-\cos^2 x = 5-\left(1-\sin^2 x\right)$ $\qquad$ $\frac{5}{2-\sqrt{x-3}}\cdot\frac{2+\sqrt{x-3}}{2+\sqrt{x-3}} = \frac{5\left(2+\sqrt{x-3}\right)}{4-(x-3)}$

- Cusp Alert!
  - If you have a variable raised to a power that is between zero and one, you will have a continuous function that is not differentiable. **The root of the term with the alerted power will be a critical value of the function!**
  - **Example**: Find the critical values of $f(x) = x^{2/3} - x$

$f(x) = (x-3)^{4/5}$

cusp/c.v. @ $x=3$

$f'(x) = \frac{2}{3}x^{-1/3} - 1 = 0$

C.V. at root of factor w/ exponent

C.V. $x=0$

$\frac{2}{3\sqrt[3]{x}} = 1$

$f'(x)=$DNE $\boxed{x=0}$

$3\sqrt[3]{x} = 2$

$\sqrt[3]{x} = \frac{2}{3}$

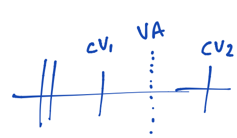
$\boxed{x = \frac{8}{27}}$

critical value

horz tangent

* In any number line chart c.v.s/p.i.v.s & any discons (VA)

1st deriv 2nd deriv



- No ~~Abbreviations~~! (max/min/IVT/EVT/MVT ok)

- Pronouns. Don't Use Pronouns. Don't be vague, ambiguous, or unclear either.
  - Don't say things like, " . . . since **it** changes from positive to negative . . .," "since **the graph** is increasing," or "**the function** changes signs **there**."
  - Be explicit. Say what you mean and mean what you say.

- Don't waste time erasing. If you draw a line through something, it becomes "invisible" to the AP graders. ~~Why So Serious?, We Are Sparta.~~

- Don't draw a line through anything on a free response or erase anything unless you have the time and intention of replacing it with something else. Something is better than nothing. Never leave anything blank, whether it's a M.C. or F.R. question. You EARN points on this test, not lose points.

- Don't let a wrong answer on one part of a F.R. question keep you from getting credit on subsequent parts. Using your wrong answer correctly, making up a reasonable equation to work with, or even attaching units to a number can get you points. NEVER give up. NEVER surrender.

- You don't need to simplify your numeric answers on the free response (unless the instructions explicitly tell you to approximate it to 3 decimals or ask you to "show" that a number equals a given number), but you MUST indicate your numeric methods to get credit. This especially goes for integral approximations from a table of values, difference quotients, and using areas from a graph to approximate integrals.

- Breathe, Relax, Smile, and get that 5!