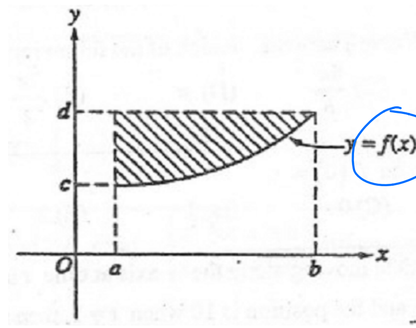
1. B	6. D
2. A	7. C
3. A	8. D
4. D	9. C
5. D	10. C

AB Review 01, No Calculator Permitted



1. Which of the following represents the area of the shaded region in the figure above?

(A) $\int_c^d f(y)\,dy$ (B) $\int_a^b (d - f(x))\,dx$ (C) $f'(b) - f'(a)$

(D) $(b-a)[f(b) - f(a)]$ (E) $(d-c)[f(b) - f(a)]$

2. If $x^3 + 3xy + 2y^3 = 17$, then in terms of x and y, $\frac{dy}{dx} =$

(A) $-\frac{x^2+y}{x+2y^2}$ (B) $-\frac{x^2+y}{x+y^2}$ (C) $-\frac{x^2+y}{x+2y}$ (D) $-\frac{x^2+y}{2y^2}$ (E) $-\frac{x^2}{1+2y^2}$

3. $\int \frac{3x^2}{\sqrt{x^3+1}}\,dx =$

(A) $2\sqrt{x^3+1}+C$ (B) $\frac{3}{2}\sqrt{x^3+1}+C$ (C) $\sqrt{x^3+1}+C$ (D) $\ln\sqrt{x^3+1}+C$ (E) $\ln(x^3+1)+C$

4. For what value of x does the function $f(x)=(x-2)(x-3)^2$ have a relative maximum?

(A) -3 (B) $-\frac{7}{3}$ (C) $-\frac{5}{2}$ (D) $\frac{7}{3}$ (E) $\frac{5}{2}$

5. If $f(x)=\sin\left(\frac{x}{2}\right)$, then there exists a number c in the interval $\frac{\pi}{2}<x<\frac{3\pi}{2}$ that satisfies the conclusion of the Mean Value Theorem. Which of the following could be c?

(A) $\frac{2\pi}{3}$ (B) $\frac{3\pi}{4}$ (C) $\frac{5\pi}{6}$ (D) π (E) $\frac{3\pi}{2}$

6. If $f(x)=(x-1)^2\sin x$, then $f'(0)=$

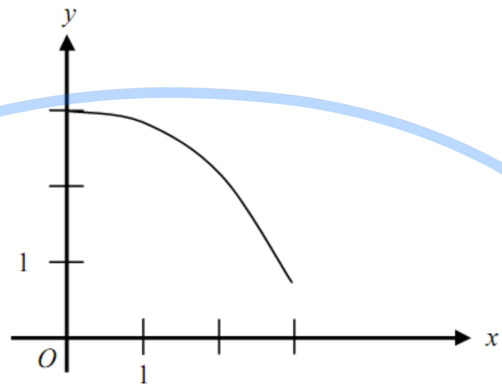
(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

7. The acceleration of a particle moving along the x-axis at time t is given by $a(t)=6t-2$. If the velocity is 25 when $t=3$ and the position is 10 when $t=1$, then the position $x(t)=$

(A) $9t^2+1$ (B) $3t^2-2t+4$ (C) t^3-t^2+4t+6 (D) $t^3-t^2+9t-20$ (E) $36t^3-4t^2-77t+55$

8. $\frac{d}{dx}\int_0^x \cos(2\pi u)\,du$ is

(A) 0 (B) $\frac{1}{2\pi}\sin x$ (C) $\frac{1}{2\pi}\cos(2\pi x)$ (D) $\cos(2\pi x)$ (E) $2\pi\cos(2\pi x)$



Graph of f

9. The graph of the function f is shown above for $0 \le x \le 3$. Of the following, which has the least value?

(A) $\int_1^3 f(x)\,dx$

(B) Left Riemann sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length.

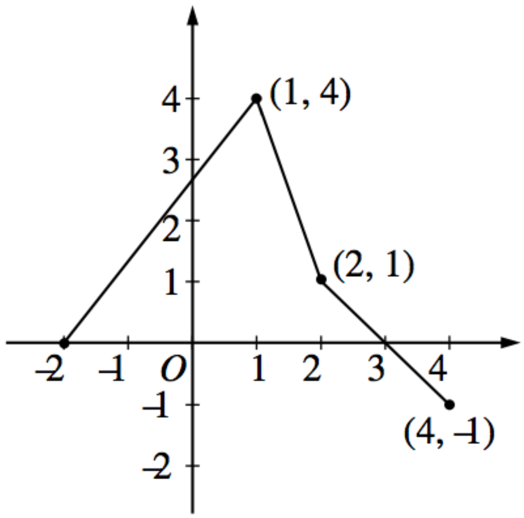
(C) Right Riemann sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length.

(D) Midpoint Riemann sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length.

(E) Trapezoidal sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length.

10. What is the minimum value of $f(x) = x\ln x$?

(A) $-e$ (B) -1 (C) $-\frac{1}{e}$ (D) 0 (E) $f(x)$ has no minimum value.



11. (1999, AB-5) The graph of the function *f*, consisting of three line segments, is shown above. Let $g(x) = \int_1^x f(t)\,dt$.

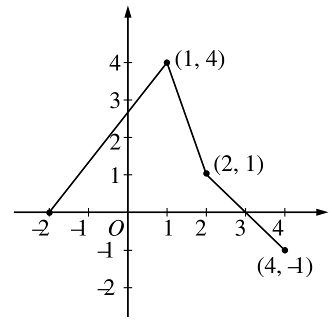
(a) Compute $g(4)$ and $g(-2)$.

(b) Find the instantaneous rate of change of *g*, with respect to *x*, at $x = 1$.

(c) Find the absolute minimum value of *g* on the closed interval $[-2, 4]$. Justify your answer.

(d) The second derivative of *g* is not defined at $x = 1$ and $x = 2$. How many of these values are *x*-coordinates of points of inflection of the graph of *g*? Justify your answer.

5. The graph of the function f, consisting of three line segments, is given above. Let $g(x) = \int_1^x f(t)\,dt$.



   (a) Compute $g(4)$ and $g(-2)$.
   (b) Find the instantaneous rate of change of g, with respect to x, at $x = 1$.
   (c) Find the absolute minimum value of g on the closed interval $[-2, 4]$. Justify your answer.
   (d) The second derivative of g is not defined at $x = 1$ and $x = 2$. How many of these values are x–coordinates of points of inflection of the graph of g? Justify your answer.

(a) $g(4) = \int_1^4 f(t)\,dt = \frac{3}{2} + 1 + \frac{1}{2} - \frac{1}{2} = \frac{5}{2}$

$g(-2) = \int_1^{-2} f(t)\,dt = -\frac{1}{2}(12) = -6$

2 { 1: $g(4)$; 1: $g(-2)$ }

(b) $g'(1) = f(1) = 4$

1: answer

(c) g is increasing on $[-2, 3]$ and decreasing on $[3, 4]$.

Therefore, g has absolute minimum at an endpoint of $[-2, 4]$.

Since $g(-2) = -6$ and $g(4) = \frac{5}{2}$,

the absolute minimum value is -6.

3 { 1: interior analysis; 1: endpoint analysis; 1: answer }

(d) One; $x = 1$

On $(-2, 1)$, $g''(x) = f'(x) > 0$

On $(1, 2)$, $g''(x) = f'(x) < 0$

On $(2, 4)$, $g''(x) = f'(x) < 0$

Therefore $(1, g(1))$ is a point of inflection and $(2, g(2))$ is not.

3 { 1: choice of $x = 1$ only; 1: show $(1, g(1))$ is a point of inflection; 1: show $(2, g(2))$ is not a point of inflection }

12. (1998, AB-4) Let f be a function with $f(1)=4$ such that for all points (x,y) on the graph of f the slope is given by $\dfrac{3x^2+1}{2y}$.

(a) Find the slope of the graph of f at the point where $x=1$.

(b) Write an equation for the line tangent to the graph of f at $x=1$, and use it to approximated $f(1.2)$.

(c) Find $f(x)$ by solving the separable differential equation $\dfrac{dy}{dx}=\dfrac{3x^2+1}{2y}$ with the initial condition $f(1)=4$.

(d) Use your solution from part (c) to find $f(1.2)$.

1998 AP Calculus AB Scoring Guidelines

4. Let f be a function with $f(1) = 4$ such that for all points (x, y) on the graph of f the slope is given by $\dfrac{3x^2+1}{2y}$.

   (a) Find the slope of the graph of f at the point where $x = 1$.

   (b) Write an equation for the line tangent to the graph of f at $x = 1$ and use it to approximate $f(1.2)$.

   (c) Find $f(x)$ by solving the separable differential equation $\dfrac{dy}{dx} = \dfrac{3x^2+1}{2y}$ with the initial condition $f(1) = 4$.

   (d) Use your solution from part (c) to find $f(1.2)$.

Solution	Scoring
(a) $\dfrac{dy}{dx} = \dfrac{3x^2+1}{2y}$ $\left.\dfrac{dy}{dx}\right\rvert_{\substack{x=1\\y=4}} = \dfrac{3+1}{2\cdot 4} = \dfrac{4}{8} = \dfrac{1}{2}$	**1**: answer
(b) $y - 4 = \dfrac{1}{2}(x-1)$ $f(1.2) - 4 \approx \dfrac{1}{2}(1.2-1)$ $f(1.2) \approx 0.1 + 4 = 4.1$	**2** { 1: equation of tangent line 1: uses equation to approximate $f(1.2)$ }
(c) $2y\,dy = (3x^2+1)\,dx$ $\int 2y\,dy = \int (3x^2+1)\,dx$ $y^2 = x^3 + x + C$ $4^2 = 1 + 1 + C$ $14 = C$ $y^2 = x^3 + x + 14$ $y = \sqrt{x^3+x+14}$ is branch with point $(1, 4)$ $f(x) = \sqrt{x^3+x+14}$	**5** { 1: separates variables 1: antiderivative of dy term 1: antiderivative of dx term 1: uses $y = 4$ when $x = 1$ to pick one function out of a family of functions 1: solves for y 0/1 if solving a linear equation in y 0/1 if no constant of integration } Note: max 0/5 if no separation of variables Note: max 1/5 [1-0-0-0-0] if substitutes value(s) for x, y, or dy/dx before antidifferentiation
(d) $f(1.2) = \sqrt{1.2^3 + 1.2 + 14} \approx 4.114$	**1**: answer, from student's solution to the given differential equation in (c)