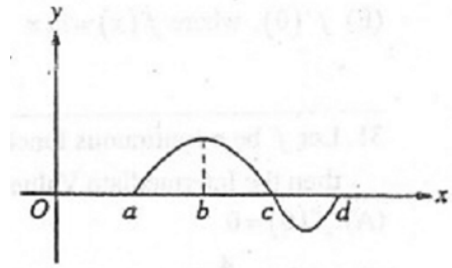
1. C
2. E
3. D
4. D
5. B
6. C
7. E
8. D
9. D
10. A

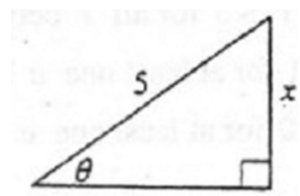
3

AB Review 03, No calculator.



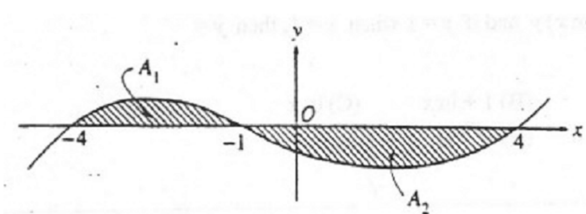
1. The graph of f is shown in the figure on the right. If $g(x)=\int_a^x f(t)\,dt$, for what value of x does $g(x)$ have a maximum?

   (A) a (B) b (C) c (D) d (E) It cannot be determined from the information given



2. In the triangle shown on the right, if θ increases at a constant rate of 3 radians per minute, at what rate is x increasing, in units per minute, when $x=3$ units?

   (A) 3 (B) $\frac{15}{4}$ (C) 4 (D) 9 (E) 12



3. The graph of $y=f(x)$ is shown in the figure above. If A_1 and A_2 are positive numbers that represent the areas of the shaded regions, then in terms of A_1 and A_2, $\int_{-4}^{4} f(x)\,dx-2\int_{-1}^{4} f(x)\,dx=$

   (A) A_1 (B) A_1-A_2 (C) $2A_1-A_2$ (D) A_1+A_2 (E) A_1+2A_2

x	0	1	2	3
$f''(x)$	5	0	-7	4

4. The polynomial function f has selected values of its second derivative f'' given in the table above. Which of the following statements must be true?

(A) f is increasing on the interval $(0,2)$.

(B) f is decreasing on the interval $(0,2)$.

(C) f has a local maximum at $x=1$.

(D) The graph of f changes concavity in the interval $(0,2)$.

5. $\displaystyle\int_1^4 \frac{\cancel{dx}\ 1}{\sqrt{16-x^2}} = dx$

$a=4 \quad u=x$

$\arcsin \frac{x}{4}\Big|_1^4$

$\arcsin 1 - \arcsin\frac{1}{4}$

$\frac{1}{4}\left(\frac{\pi}{2} - \arcsin\frac{1}{4}\right)$

(A) $\arcsin\left(\frac{1}{4}\right)+\frac{\pi}{2}$ (B) $-\arcsin\left(\frac{1}{4}\right)+\frac{\pi}{2}$ (C) $\arcsin\left(\frac{1}{4}\right)-\frac{\pi}{2}$

(D) $-4\arcsin\left(\frac{1}{4}\right)+\frac{\pi}{2}$ (E) $4\arcsin\left(\frac{1}{4}\right)-\frac{\pi}{2}$

6. $\displaystyle\int \frac{\cancel{x}\ 1}{(x^2-4)}\,dx =$ $\frac{1}{2}\ln|x^2-4|+c$

(A) $\dfrac{-1}{4\left(x^2-4\right)^2}+C$ (B) $\dfrac{1}{2\left(x^2-4\right)}+C$ (C) $\dfrac{1}{2}\ln\left|x^2-4\right|+C$ (D) $2\ln\left|x^2-4\right|+C$ (E) $\dfrac{1}{2}\arctan\left(\dfrac{x}{2}\right)+C$

7. The position of a particle moving along the x-axis at time t is given by $x(t)=\sin^2(4\pi t)$. At which of the following values of t will the particle change direction?

I. $t=\frac{1}{8}$

II. $t=\frac{1}{6}$

III. $t=1$

IV. $t=2$

(A) II, III, and IV (B) I and II (C) I, II, and III (D) III and IV (E) I, III, and IV

8. If $\sin(xy)=x$, then $\frac{dy}{dx}=$

(A) $\frac{1}{\cos(xy)}$ (B) $\frac{1}{x\cos(xy)}$ (C) $\frac{1-\cos(xy)}{\cos(xy)}$ (D) $\frac{1-y\cos(xy)}{x\cos(xy)}$ (E) $\frac{y(1-\cos(xy))}{x}$

$f'(0)=20$

$f'(x)=4x+20e^{5x}$ $f:(0,4)$ $f^{-1}:(4,0)$

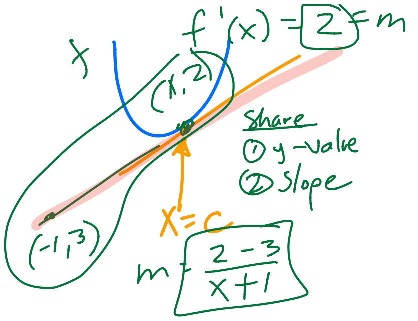
9. The function $f(x)=2x^2+4e^{5x}$ has an inverse function $f^{-1}(x)$. Find the slope of the normal line to the graph of $f^{-1}(x)$ at $x=f(0)=4$

(A) $16+20e^{20}$ (B) $\frac{1}{20}=(f^{-1})'$ (C) $-\frac{1}{16+20e^{20}}$ (D) $\boxed{-20}$ (E) $-\frac{5}{4}$

10. In the xy-plane, the line $x+y=k$, where k is a constant, is tangent to the graph of $y=\boxed{x^2+3x+1}$. What is the value of k?

$y=\boxed{-x+k}$

(A) -3 (B) -2 (C) -1 (D) 0 (E) 1



y-vales: $x^2+3x+1=-x+k$

Slopes: $y'=2x+3$

$y'=-1$

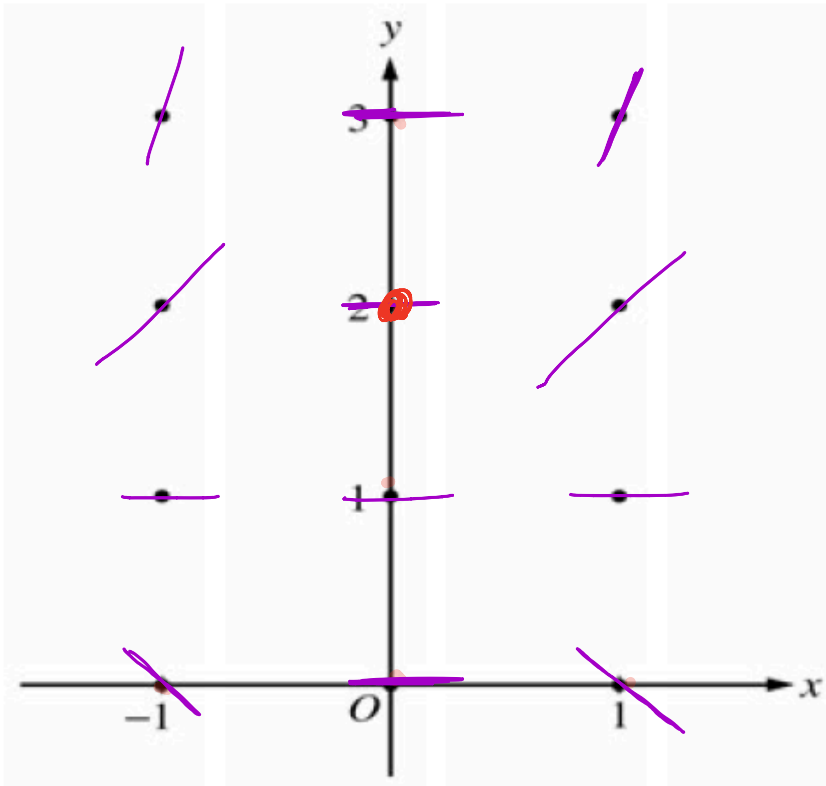
$2x+3=-1$

$2x=-4$

$\boxed{x=-2}$

$4-6+1=2+k$

$\boxed{-3=k}$



11. (2004, AB-6) Consider the differential equation given by $\frac{dy}{dx} = x^2(y-1)$. $= 0$

(a) On the axes provided above, sketch a slope field for the given differential equation at the 12 points indicated.

(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the *xy*-plane. Describe all points in the *xy*-plane for which the slopes are positive.

$$\frac{dy}{dx} = x^2(y-1) > 0 \qquad y-1>0,\quad y>1$$

The slopes are positive for all $y>1$, $x \neq 0$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$.

$$\frac{dy}{dx} = x + y$$

$$\frac{dy}{dx} = x^2(y-1) = x^2y - x^2 = \boxed{x^2(y-1)}$$

(√1) $\int \frac{1}{y-1}\,dy = \int x^2\,dx$ (√3) (√4)

(√2) $\ln|y-1| = \frac{1}{3}x^3 + C$

e e

$|y-1| = e^{(x^3/3 + C)}$

$y - 1 = Ce^{x^3/3}$

Gen: $\boxed{y = 1 + Ce^{x^3/3}}$ (√5)

At $(0,3)$: $3 = 1 + Ce^0$

$\boxed{2 = C}$

So $\boxed{y = 1 + 2e^{x^3/3}}$ $\sqrt{6}$

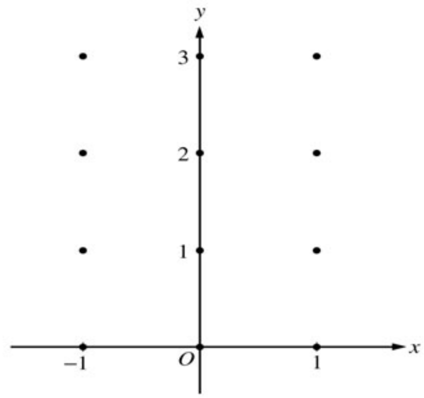
Question 6

Consider the differential equation $\frac{dy}{dx} = x^2(y-1)$.

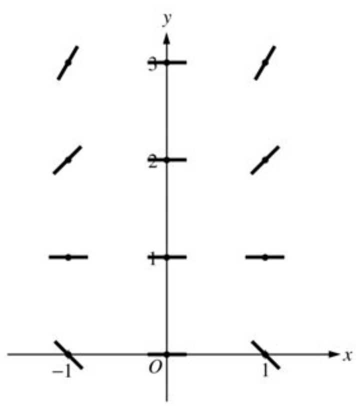
(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.
(**Note: Use the axes provided in the pink test booklet.**)

(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane. Describe all points in the xy-plane for which the slopes are positive.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$.



(a)



$2 : \begin{cases} 1 : \text{zero slope at each point } (x, y) \text{ where } x = 0 \text{ or } y = 1 \\ 1 : \begin{cases} \text{positive slope at each point } (x, y) \text{ where } x \neq 0 \text{ and } y > 1 \\ \text{negative slope at each point } (x, y) \text{ where } x \neq 0 \text{ and } y < 1 \end{cases} \end{cases}$

(b) Slopes are positive at points (x, y) where $x \neq 0$ and $y > 1$.

1 : description

(c)
$$\frac{1}{y-1}\,dy = x^2\,dx$$
$$\ln|y-1| = \frac{1}{3}x^3 + C$$
$$|y-1| = e^C e^{\frac{1}{3}x^3}$$
$$y - 1 = Ke^{\frac{1}{3}x^3},\ K = \pm e^C$$
$$2 = Ke^0 = K$$
$$y = 1 + 2e^{\frac{1}{3}x^3}$$

$6 : \begin{cases} 1 : \text{separates variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \\ \quad 0/1 \text{ if } y \text{ is not exponential} \end{cases}$

Note: max $3/6$ [1-2-0-0-0] if no constant of integration

Note: $0/6$ if no separation of variables

12. (1999, AB-4) Suppose that the function f has a continuous second derivative for all x, and that $f(0)=2$, $f'(0)=-3$, and $f''(0)=0$. Let g be a function whose derivative is given by

$g'(x)=e^{-2x}\left(3f(x)+2f'(x)\right)$ for all x.

(a) Write an equation of the tangent line to the graph of f at the point where $x=0$.

$g'(0)=1\cdot\left(3f(0)+2f'(0)\right)$
$=3(2)+2(-3)$
$=6-6$
$=0$

y-value: $f(0)=2$
m: $f'(0)=-3$
eq: $\boxed{y=2-3(x-0)}$

(b) Is there sufficient information to determine whether or not the graph of f has a point of inflection when $x=0$? Explain your answer.

↳ ① $f''=0$ or $f''=\text{DNE} \to$ p.i.v.s
② sign change at the p.i.v. in $f''(x)$

$f''(0)=0$, $x=0$ is a p.i.v.

x	-1	0	1
f''			

No! We don't have $f''(x)$ information on either side of $x=0$.

(c) Given that $g(0)=4$, write an equation of the line tangent to the graph of g at the point where $x=0$.

$g(0)=4$
$g'(0)=0$
$\boxed{y=4+0(x-0)}$
or $\boxed{y=4}$

(d) Show that $g''(x)=e^{-2x}\left(-6f(x)-f'(x)+2f''(x)\right)$. Does g have a local maximum at $x=0$? Justify your answer.

$g'(x)=e^{-2x}\left(3f(x)+2f'(x)\right)$

$g''(x)=-2e^{-2x}\left(3f(x)+2f'(x)\right)+e^{-2x}\left(3\cdot f'(x)+2f''(x)\right)$

$=e^{-2x}\left[-6f(x)-4f'(x)+3f'(x)+2f''(x)\right]$

$=e^{-2x}\left[-6f(x)-f'(x)+2f''(x)\right]$

g has a local max at $x=0$

$\boxed{g'(0)=0}$

$g''(0)=1\left[-6f(0)-f'(0)+2f''(0)\right]$
$=-12+3$

1st

x

$g'(x)$

0 1

2nd

$g''(0) > 0$

$g''(0) < 0$

$= -9 < 0$ so g has a local max @ $x = 0$

AB–4 1999

4. Suppose that the function f has a continuous second derivative for all x, and that $f(0) = 2$, $f'(0) = -3$, and $f''(0) = 0$. Let g be a function whose derivative is given by $g'(x) = e^{-2x}(3f(x) + 2f'(x))$ for all x.
   (a) Write an equation of the line tangent to the graph of f at the point where $x = 0$.
   (b) Is there sufficient information to determine whether or not the graph of f has a point of inflection when $x = 0$? Explain your answer.
   (c) Given that $g(0) = 4$, write an equation of the line tangent to the graph of g at the point where $x = 0$.
   (d) Show that $g''(x) = e^{-2x}(-6f(x) - f'(x) + 2f''(x))$. Does g have a local maximum at $x = 0$? Justify your answer.

(a) Slope at $x = 0$ is $f'(0) = -3$

At $x = 0$, $y = 2$

$y - 2 = -3(x - 0)$

1: equation

(b) No. Whether $f''(x)$ changes sign at $x = 0$ is unknown. The only given value of $f''(x)$ is $f''(0) = 0$.

2 $\begin{cases} 1\text{: answer} \\ 1\text{: explanation} \end{cases}$

(c) $g'(x) = e^{-2x}(3f(x) + 2f'(x))$

$$g'(0) = e^{0}(3f(0) + 2f'(0))$$
$$= 3(2) + 2(-3) = 0$$

$y - 4 = 0(x - 0)$

$y = 4$

2 $\begin{cases} 1\text{: } g'(0) \\ 1\text{: equation} \end{cases}$

(d) $g'(x) = e^{-2x}(3f(x) + 2f'(x))$

$$g''(x) = (-2e^{-2x})(3f(x) + 2f'(x)) + e^{-2x}(3f'(x) + 2f''(x))$$
$$= e^{-2x}(-6f(x) - f'(x) + 2f''(x))$$

$g''(0) = e^{0}[(-6)(2) - (-3) + 2(0)] = -9$

Since $g'(0) = 0$ and $g''(0) < 0$, g does have a local maximum at $x = 0$.

4 $\begin{cases} 2\text{: verify derivative} \\ \quad 0/2 \text{ product or chain rule error} \\ \quad <-1> \text{ algebra errors} \\ 1\text{: } g'(0) = 0 \text{ and } g''(0) \\ 1\text{: answer and reasoning} \end{cases}$