for Monday's skippers

1. E	6. E
2. D	7. C
3. D	8. A
4. A	9. E
5. D	10. D

6

AB Review 06, No Calculator Permitted, except on #11 & #12

1. Let f and g be differentiable functions with the following properties:

   (i) $g(x) > 0$ for all x

   (ii) $f(0) = 1$ $y=1$

   If $h(x) = f(x)g(x)$ and $h'(x) = f(x)g'(x)$, then $f(x) =$

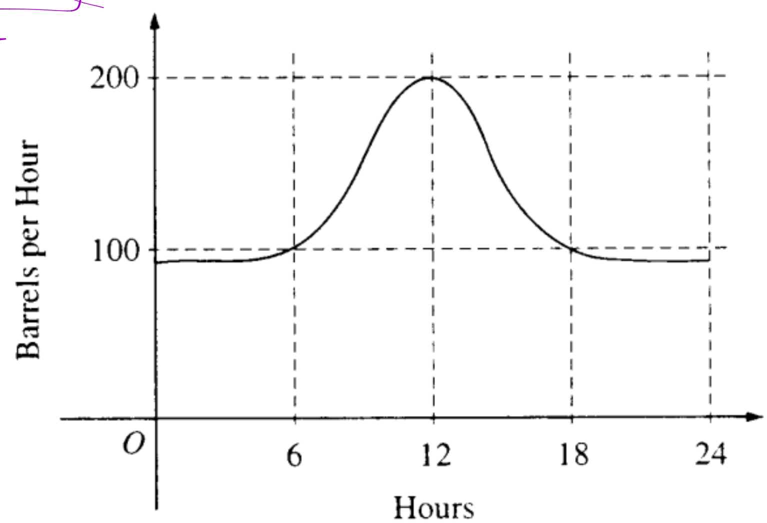
   $h'(x) = f'(x)g(x) + \boxed{f(x)g'(x)}$

   (A) $f'(x)$ (B) $g(x)$ (C) e^x (D) 0 (E) 1

   $f'(x)g(x) = 0$

   $f'(x) = 0$ or $\cancel{g(x) = 0}$

   f is a constant



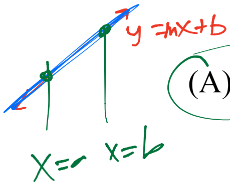
2. The flow of oil, in barrels per hour, through a pipeline on July 9 is given by the graph above. Of the following, which best approximates the total number of barrels of oil that passed through the pipeline that day?

   (A) 500 (B) 600 (C) 2,400 (D) 3,000 (E) 4,800

3. What is the instantaneous rate of change at $x = 2$ of the function f given by $f(x) = \dfrac{x^2 - 2}{x - 1}$?

   (A) -2 (B) $\dfrac{1}{6}$ (C) $\dfrac{1}{2}$ (D) 2 (E) 6

4. If f is a linear function and $0 < a < b$, then $\int_a^b f''(x)\,dx =$ $f'\Big|_a^b = f'(b) - f'(a) = 0$



(A) 0 (B) 1 (C) $\dfrac{ab}{2}$ (D) $b-a$ (E) $\dfrac{b^2-a^2}{2}$

5. If $F(x) = \int_0^x \sqrt{t^3+1}\,dt$, then $F'(2) =$

(A) -3 (B) -2 (C) 2 (D) 3 (E) 18

6. If $f(x) = \sin\left(e^{-x}\right)$, then $f'(x) =$

(A) $-\cos\left(e^{-x}\right)$ (B) $\cos\left(e^{-x}\right)+e^{-x}$ (C) $\cos\left(e^{-x}\right)-e^{-x}$ (D) $e^{-x}\cos\left(e^{-x}\right)$ (E) $-e^{-x}\cos\left(e^{-x}\right)$

7. If $f''(x) = x(x+1)(x-2)^2$, then the graph of f has inflection points when $x =$

(A) -1 only (B) 2 only (C) -1 and 0 only (D) -1 and 2 only (E) -1, 0, and 2 only

8. What are all the values of k for which $\int_{-3}^{k} x^2 dx = 0$?

(A) -3 (B) 0 (C) 3 (D) -3 and 3 (E) -3, 0, and 3

9. The average value of the function $f(x) = 2e^{(x-3)}$ on the interval $[1,6]$ is

(A) $\frac{e^3}{3}$ (B) $2e^3 - 2e^{-2}$ (C) $\frac{e^3}{3} - \frac{e^2}{3}$ (D) $e^3 + e^{-5}$ (E) $\frac{2e^3}{5} - \frac{2e^{-2}}{5}$

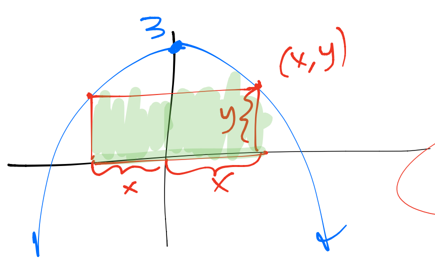
$$\frac{\int_1^6 2e^{(x-3)}\,dx}{6-1} = \frac{2}{5}e^{x-3}\Big|_1^6$$

$$\frac{2}{5}\left[e^3 - e^{-2}\right]$$

10. A rectangle has its base on the x-axis and both its other vertices on the positive portion of the parabola $y = 3 - 4x^2$. What is the maximum possible area of this rectangle?

optimization.

(A) $\frac{3\sqrt{6}}{4}$ (B) $\frac{3\sqrt{15}}{5}$ (C) $\frac{3\sqrt{15}}{10}$ (D) 2 (E) $\frac{3}{2}$



$A = 2xy$

$A = 2x(3-4x^2)$

$A = 6x - 8x^3$

$A' = 6 - 24x^2 = 0$

$6 = 24x^2$

$x^2 = \frac{1}{4}$

$x = \pm\frac{1}{2}$

$x = \frac{1}{2}$

$\text{Area} = 3 - 4\left(\frac{1}{4}\right)$

$= 3 - 1$

$= \boxed{2}$

11. (Calculator Permitted) (2003, AB-2) A particle moves along the x-axis so that its velocity at time t is given by $v(t) = -(t+1)\sin\left(\frac{t^2}{2}\right)$. At time $t = 0$, the particle is at position $x = 1$.

initial condition

(a) Find the acceleration of the particle at time $t = 2$. Is the speed of the particle increasing at $t = 2$? Why or why not?

$\int_a^b v(t)\,dt \to$ Disp

$\int_a^b |v(t)|\,dt \to$ Dist Travel

$x(b) = x(a) + \int_a^b v(t)\,dt$

$v'(2) = a(2) = 1.587$

$v(2) = -2.727$

The speed is decreasing at $t=2$, since $a(2) > 0$ and $v(2) < 0$.

(b) Find all times t in the open interval $0 < t < 3$ when the particle changes direction. Justify your answer.

v(t) change signs

$v(t) = 0$

$t = 2.506$

the particle changes directions at $t = 2.506$ since $v(t)$ changes from negative to positive at $t = 2.506$.

(c) Find the total distance traveled by the particle from time $t = 0$ until time $t = 3$.

$\text{Total Distance} = \int_0^3 |v(t)|\,dt$

$= 4.333$

(d) During the time interval $0 \le t \le 3$, what is the greatest distance between the particle and the origin? Show the work that leads to your answer.

$x(t) = \text{Position}$

$x'(t) = v(t) = 0$

$x(0) = 1$

$x(3) = 1 + \underbrace{\int_0^3 v(t)\,dt}_{\text{Disp}} = -1.197$

$x(2.506) = 1 + \int_0^{2.506} v(t)\,dt = -2.265$

EVT

So, the greatest distance is 2.265.

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Question 2

A particle moves along the x-axis so that its velocity at time t is given by

$$v(t) = -(t+1)\sin\left(\frac{t^2}{2}\right).$$

At time $t = 0$, the particle is at position $x = 1$.

(a) Find the acceleration of the particle at time $t = 2$. Is the speed of the particle increasing at $t = 2$? Why or why not?

(b) Find all times t in the open interval $0 < t < 3$ when the particle changes direction. Justify your answer.

(c) Find the total distance traveled by the particle from time $t = 0$ until time $t = 3$.

(d) During the time interval $0 \le t \le 3$, what is the greatest distance between the particle and the origin? Show the work that leads to your answer.

(a) $a(2) = v'(2) = 1.587$ or 1.588

$v(2) = -3\sin(2) < 0$

Speed is decreasing since $a(2) > 0$ and $v(2) < 0$.

2 : { 1 : $a(2)$
1 : speed decreasing with reason }

(b) $v(t) = 0$ when $\frac{t^2}{2} = \pi$

$t = \sqrt{2\pi}$ or 2.506 or 2.507

Since $v(t) < 0$ for $0 < t < \sqrt{2\pi}$ and $v(t) > 0$ for $\sqrt{2\pi} < t < 3$, the particle changes directions at $t = \sqrt{2\pi}$.

2 : { 1 : $t = \sqrt{2\pi}$ only
1 : justification }

(c) Distance $= \int_0^3 |v(t)|\,dt = 4.333$ or 4.334

3 : { 1 : limits
1 : integrand
1 : answer }

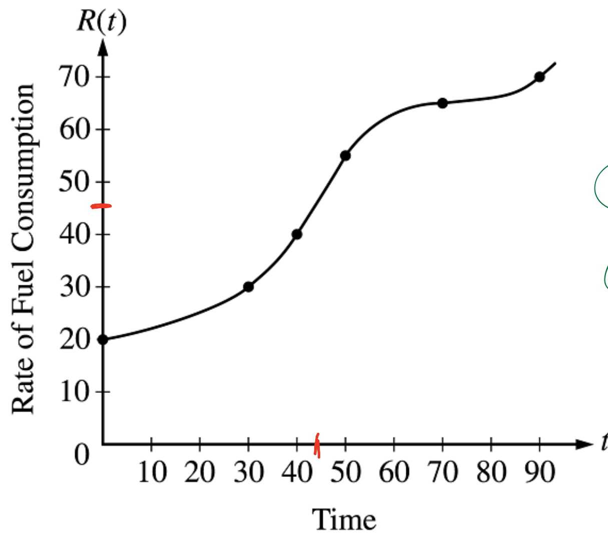
(d) $\int_0^{\sqrt{2\pi}} v(t)\,dt = -3.265$

$x\left(\sqrt{2\pi}\right) = x(0) + \int_0^{\sqrt{2\pi}} v(t)\,dt = -2.265$

Since the total distance from $t = 0$ to $t = 3$ is 4.334, the particle is still to the left of the origin at $t = 3$. Hence the greatest distance from the origin is 2.265.

Considers endpts

2 : { ~~1 : ± (distance particle travels~~
~~while velocity~~ is negative)
1 : answer }



t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

30, 10, 10, 20, 20

$\int_0^{90} R(t)\,dt \approx (30)(20) + (10)(30) + \cdots$ gal

12. (Calculator Permitted) (2003, AB-3) The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function R of time t. The graph of R and a table of selected values of $R(t)$, for the time interval $0 \le t \le 90$ minutes, are shown above.

(a) Use data from the table to find an approximation for $R'(45)$. Show the computations that lead to your answer. Indicate units of measure.

$R'(45) \approx \dfrac{55-40}{50-40}$ gal/min²

R(t) R'(45) is A MAX

(b) The rate of fuel consumption is increasing fastest at time $t = 45$ minutes. What is the value of $\boxed{R''(45)}$? Explain your reasoning.

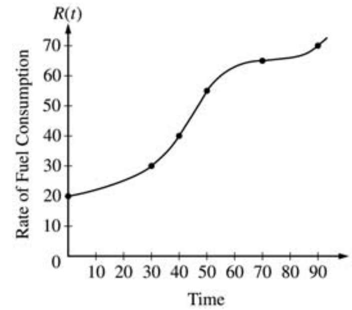
$R''(45) = 0$ ✓
Since → R has an inflection value @ t=45min → $R'(45)$ is a maximum

(c) Approximate the value of $\int_0^{90} R(t)\,dt$ using a left Riemann sum with five subintervals indicated by the data in the table. Is this numerical approximation less than the value of $\int_0^{90} R(t)\,dt$? Explain your reasoning.

(d) For $0 < b \le 90$ minutes, explain the meaning of $\int_0^{b} R(t)\,dt$ in terms of fuel consumption for the plane. Explain the meaning of $\frac{1}{b}\int_0^{b} R(t)\,dt$ in terms of fuel consumption for the plane. Indicate units of measure in both answers.

Question 3

The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function R of time t. The graph of R and a table of selected values of $R(t)$, for the time interval $0 \le t \le 90$ minutes, are shown above.



t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

(a) Use data from the table to find an approximation for $R'(45)$. Show the computations that lead to your answer. Indicate units of measure.

(b) The rate of fuel consumption is increasing fastest at time $t = 45$ minutes. What is the value of $R''(45)$? Explain your reasoning.

(c) Approximate the value of $\int_0^{90} R(t)\,dt$ using a left Riemann sum with the five subintervals indicated by the data in the table. Is this numerical approximation less than the value of $\int_0^{90} R(t)\,dt$? Explain your reasoning.

(d) For $0 < b \le 90$ minutes, explain the meaning of $\int_0^{b} R(t)\,dt$ in terms of fuel consumption for the plane. Explain the meaning of $\frac{1}{b}\int_0^{b} R(t)\,dt$ in terms of fuel consumption for the plane. Indicate units of measure in both answers.

(a) $R'(45) \approx \dfrac{R(50) - R(40)}{50 - 40} = \dfrac{55 - 40}{10}$
$= 1.5 \text{ gal/min}^2$

2 : { 1 : a difference quotient using numbers from table and interval that contains 45
1 : 1.5 gal/min^2 }

(b) $R''(45) = 0$ since $R'(t)$ has a maximum at $t = 45$.

2 : { 1 : $R''(45) = 0$
1 : reason }

(c) $\int_0^{90} R(t)\,dt \approx (30)(20) + (10)(30) + (10)(40)$
$+ (20)(55) + (20)(65) = 3700$
Yes, this approximation is less because the graph of R is increasing on the interval.

2 : { 1 : value of left Riemann sum
1 : "less" with reason }

(d) $\int_0^{b} R(t)\,dt$ is the total amount of fuel in gallons consumed for the first b minutes.
$\frac{1}{b}\int_0^{b} R(t)\,dt$ is the average value of the rate of fuel consumption in gallons/min during the first b minutes.

3 : { 2 : meanings
1 : meaning of $\int_0^{b} R(t)\,dt$
1 : meaning of $\frac{1}{b}\int_0^{b} R(t)\,dt$
< −1 > if no reference to time b
1 : units in both answers }