1. A
2. D
3. B
4. B
5. A
6. D
7. B
8. D
9. C
10. B

8

AB Review 08 Calculator Permitted (unless stated otherwise)

1. $\lim_{h \to 0} \frac{\ln(e+h)-1}{h}$ is

(A) $f'(e)$, where $f(x)=\ln x$ (B) $f'(e)$, where $f(x)=\frac{\ln x}{x}$ (C) $f'(1)$, where $f(x)=\ln x$

(D) $f'(1)$, where $f(x)=\ln(x+e)$ (E) $f'(0)$, where $f(x)=\ln x$

2. The position of an object attached to a spring is given by $y(t)=\frac{1}{6}\cos(5t)-\frac{1}{4}\sin(5t)$, where t is time in seconds. In the first 4 seconds, how many times is the velocity of the object equal to 0?

(A) Zero (B) Three (C) Five (D) Six (E) Seven

3. Let f be the function given by $f(x)=\cos(2x)+\ln(3x)$. What is the least value of x at which the graph of f changes concavity?

(A) 0.56 (B) 0.93 (C) 1.18 (D) 2.38 (E) 2.44

4. If $0 \le x \le 4$, of the following, which is the greatest value of x such that $\int_0^x (t^2-2t)\,dt \ge \int_2^x t\,dt$?

(A) 1.35 (B) 1.38 (C) 1.41 (D) 1.48 (E) 1.59

5. Let f be a differentiable function such that $f(3)=15$, $f(6)=3$, $f'(3)=-8$, and $f'(6)=-2$. The function g is differentiable and $g(x)=f^{-1}(x)$ for all x. What is the value of $g'(3)$?

(A) $-\frac{1}{2}$ (B) $-\frac{1}{8}$ (C) $\frac{1}{6}$ (D) $\frac{1}{3}$

(E) The value of $g'(3)$ cannot be determined from the information given

$g: (3, 6)$

$f: (6, 3)$

$$g'(3) = \frac{1}{f'(6)}$$

6. Let f be a continuous function on the closed interval $[-3,6]$. If $f(-3)=-1$ and $f(6)=3$, then the Intermediate Value Theorem guarantees that

(A) $f(0)=0$

(B) $f'(c)=\frac{4}{9}$ for at least one c between -3 and 6

(C) $-1 \le f(x) \le 3$ for all x between -3 and 6

(D) $f(c)=1$ for at least one c between -3 and 6

(E) $f(c)=0$ for at least one c between -1 and 3

7. The first derivative of the function f is defined by $f'(x)=\sin\left(x^3-x\right)$ for $0 \le x \le 2$. On what interval(s) is f increasing?

(A) $1 \le x \le 1.445$ (B) $1 \le x \le 1.691$ (C) $1.445 \le x \le 1.875$

(D) $0.577 \le x \le 1.445$ and $1.875 \le x \le 2$ (E) $0 \le x \le 1$ and $1.691 \le x \le 2$

8. Let f be the function given by $f(x) = \int_{1/3}^{x} \cos\left(\frac{1}{t^2}\right) dt$ for $\frac{1}{3} \le t \le 1$. At which of the following values of x does f attain a relative maximum?

(A) 0.357 and 0.798 (B) 0.4 and 0.564 (C) 0.4 only (D) 0.461 (E) 0.999

① $f' = 0$ or $f' = DNE$
c.v.
② sign of f' to change from $\underline{+}$ to $\underline{-}$

$f' = \cos\left(\frac{1}{x^2}\right)$

9. Let f be the function defined by $f(x) = x + \ln x$. What is the value of c for which the instantaneous rate of change of f at $x = c$ is the same as the average rate of change of f over $[1, 4]$?

(A) 0.456 (B) 1.244 (C) 2.164 (D) 2.342 (E) 2.452

10. The height, h, in meters, of an object at time t is given by $h(t) = 24t + 24t^{3/2} - 16t^2$. What is the height of the object at the instant when it reaches its maximum upward velocity?

(A) 2.545 meters (B) 10.263 meters (C) 34.125 meters (D) 54.889 meters (E) 89.005 meters

11. (2012, AB-5) (No Calculator) The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0$, when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after it is first weighed, then

$$\boxed{\frac{dB}{dt} = \frac{1}{5}(100 - B).} \quad \text{gram/day}$$

Let $y = B(t)$ be the solution to the differential equation above with the initial condition $\boxed{B(0) = 20.}$

(a) Is the bird gaining weight faster when it weighs 40 grams or when it weighs 70 grams? Explain your reasoning.

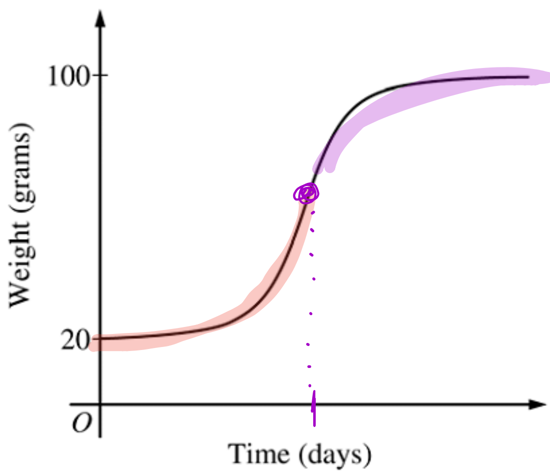
$5\overline{)60} = 12$; 5, 10, 10, 0

$$\left.\frac{dB}{dt}\right|_{B=40} = \frac{1}{5}(100-40) = 12 \text{ gram/day}$$

$$\left.\frac{dB}{dt}\right|_{B=70} = \frac{1}{5}(100-70) = 6 \text{ gram/day}$$

So the bird is gaining weight faster when it weighs 40grams.

(b) Find $\boxed{\frac{d^2B}{dt^2} \text{ in terms of } B}$. Use $\frac{d^2B}{dt^2}$ to explain why the graph of B cannot resemble the following graph.



$$\frac{d}{dt}\left[\frac{dB}{dt}\right] = \frac{d}{dt}\left[\frac{1}{5}(100-B)\right]$$

$$\frac{d^2B}{dt^2} = \frac{1}{5}\left(-\frac{dB}{dt}\right)$$

$$\frac{d^2B}{dt^2} = -\frac{1}{5}\left(\frac{1}{5}(100-B)\right) = -\frac{1}{25}(100-B)$$

the graph of B(t) should be concave down to start with but the given graph is concave up to start with, so it cannot be the graph of B(t).

(c) Use separation of variables to find $y = B(t)$, the particular solution to the differential equation with initial condition $B(0) = 20$.

$$\frac{dB}{dt} = \frac{1}{5}(100 - B)$$

$$\int \frac{1}{(100-B)} dB = \int \frac{1}{5} dt \checkmark$$

$$-\ln|100-B| = \frac{1}{5}t + C \checkmark$$

$$\ln|100-B| = -\frac{1}{5}t + C$$

$$100 - B = Ce^{-t/5}$$

$$B = 100 + Ce^{-t/5}$$

At $(0,20)$ For $B(0)=20$: $20 = 100 + C$ $\checkmark$

$$\boxed{C = -80} \checkmark$$

So $\boxed{B(t) = 100 - 80e^{-t/5}}$

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Question 5

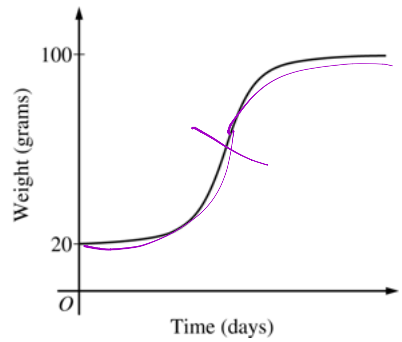
The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0$, when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after it is first weighed, then

$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

Let $y = B(t)$ be the solution to the differential equation above with initial condition $B(0) = 20$.

(a) Is the bird gaining weight faster when it weighs 40 grams or when it weighs 70 grams? Explain your reasoning.

(b) Find $\frac{d^2B}{dt^2}$ in terms of B. Use $\frac{d^2B}{dt^2}$ to explain why the graph of B cannot resemble the following graph.



(c) Use separation of variables to find $y = B(t)$, the particular solution to the differential equation with initial condition $B(0) = 20$.

(a) $\left.\frac{dB}{dt}\right|_{B=40} = \frac{1}{5}(60) = 12$

$\left.\frac{dB}{dt}\right|_{B=70} = \frac{1}{5}(30) = 6$

Because $\left.\frac{dB}{dt}\right|_{B=40} > \left.\frac{dB}{dt}\right|_{B=70}$, the bird is gaining weight faster when it weighs 40 grams.

$2 : \begin{cases} 1 : \text{uses } \frac{dB}{dt} \\ 1 : \text{answer with reason} \end{cases}$

(b) $\frac{d^2B}{dt^2} = -\frac{1}{5}\frac{dB}{dt} = -\frac{1}{5}\cdot\frac{1}{5}(100 - B) = -\frac{1}{25}(100 - B)$

~~Therefore, the graph of B is concave down for~~ $\boxed{20 \le B < 100.}$ ~~A portion of the given graph is concave up.~~

$2 : \begin{cases} 1 : \frac{d^2B}{dt^2} \text{ in terms of } B \\ 1 : \text{explanation} \end{cases}$

(c) $\frac{dB}{dt} = \frac{1}{5}(100 - B)$

$\int \frac{1}{100 - B}\, dB = \int \frac{1}{5}\, dt$

$-\ln|100 - B| = \frac{1}{5}t + C$

Because $20 \le B < 100$, $|100 - B| = 100 - B$.

$-\ln(100 - 20) = \frac{1}{5}(0) + C \Rightarrow -\ln(80) = C$

$100 - B = 80e^{-t/5}$

$B(t) = 100 - 80e^{-t/5}, \quad t \ge 0$

$5 : \begin{cases} 1 : \text{separation of variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } B \end{cases}$

Note: max $2/5$ [1-1-0-0-0] if no constant of integration

Note: $0/5$ if no separation of variables

12. (2011, AB-1) (Calculator Permitted) For $0 \le t \le 6$, a particle is moving along the x-axis. The particle's position, $x(t)$, is not explicitly given. The velocity of the particle is given by $\boxed{v(t) = 2\sin\left(e^{t/4}\right) + 1}$. The acceleration of the particle is given by $\boxed{a(t) = \frac{1}{2}e^{t/4}\cos\left(e^{t/4}\right)}$ and $x(0) = 2$.

(a) Is the speed of the particle increasing or decreasing at time $t = 5.5$? Give a reason for your answer.

$v(5.5) = -0.453 < 0$

$a(5.5) = -1.358 < 0$

So speed is increasing at $t = 5.5$.

(b) Find the average velocity of the particle for the time period $0 \le t \le 6$.

$$\text{Avg vel} = \frac{\int_0^6 v(t)\,dt}{6-0}$$

$$= 1.949$$

(c) Find the total distance traveled by the particle from time $t = 0$ to $t = 6$.

$$\text{Dist} = \int_0^6 |v(t)|\,dt$$

$$= 12.573$$

(d) For $0 \le t \le 6$, the particle changes direction exactly once. Find the position of the particle at that time.

$$X(A) = 2 + \int_0^A v(t)\,dt$$

Question 1

For $0 \le t \le 6$, a particle is moving along the x-axis. The particle's position, $x(t)$, is not explicitly given. The velocity of the particle is given by $v(t) = 2\sin\left(e^{t/4}\right) + 1$. The acceleration of the particle is given by $a(t) = \frac{1}{2}e^{t/4}\cos\left(e^{t/4}\right)$ and $x(0) = 2$.

(a) Is the speed of the particle increasing or decreasing at time $t = 5.5$? Give a reason for your answer.

(b) Find the average velocity of the particle for the time period $0 \le t \le 6$.

(c) Find the total distance traveled by the particle from time $t = 0$ to $t = 6$.

(d) For $0 \le t \le 6$, the particle changes direction exactly once. Find the position of the particle at that time.

(a) $v(5.5) = -0.45337,\ a(5.5) = -1.35851$

The speed is increasing at time $t = 5.5$, because velocity and acceleration have the same sign.

2 : conclusion with reason

(b) Average velocity $= \frac{1}{6}\int_0^6 v(t)\,dt = 1.949$

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(c) Distance $= \int_0^6 |v(t)|\,dt = 12.573$

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(d) $v(t) = 0$ when $t = 5.19552$. Let $b = 5.19552$.
$v(t)$ changes sign from positive to negative at time $t = b$.
$x(b) = 2 + \int_0^b v(t)\,dt = 14.134$ or 14.135

$3 : \begin{cases} 1 : \text{considers } v(t) = 0 \\ 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$