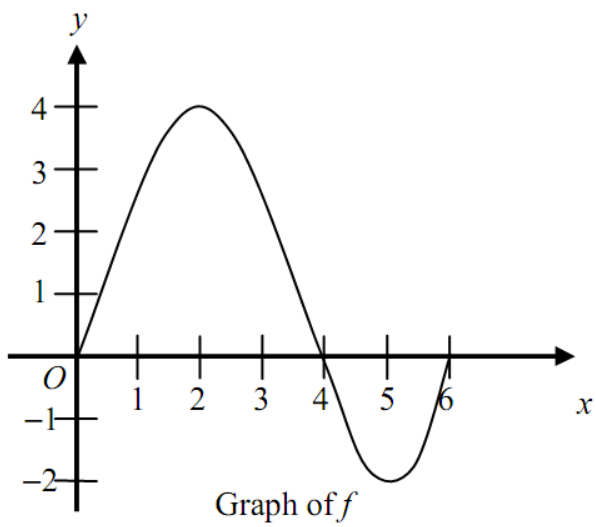
1. A	6. B	10
2. C	7. C	
3. E	8. B	
4. B	9. E	
5. C	10. C	

1.

If $f(x) = (\ln x)^2$, then $f''(\sqrt{e}) =$

(A) $\frac{1}{e}$ (B) $\frac{2}{e}$ (C) $\frac{1}{2\sqrt{e}}$ (D) $\frac{1}{\sqrt{e}}$ (E) $\frac{2}{\sqrt{e}}$

2.



Graph of f

The graph of the function f shown above has horizontal tangents at $x = 2$ and $x = 5$. Let g be the function defined by $g(x) = \int_0^x f(t)\,dt$. For what values of x does the graph of g have a point of inflection?

(A) 2 only (B) 4 only (C) 2 and 5 only (D) 2, 4, and 5 (E) 0, 4, and 6

3.

Which of the following is the solution to the differential equation $\frac{dy}{dx} = \frac{x^2}{y}$ with the initial condition $y(3) = -2$?

(A) $y = 2e^{-9+x^3/3}$

(B) $y = -2e^{-9+x^3/3}$

(C) $y = \sqrt{\frac{2x^3}{3}}$

(D) $y = \sqrt{\frac{2x^3}{3} - 14}$

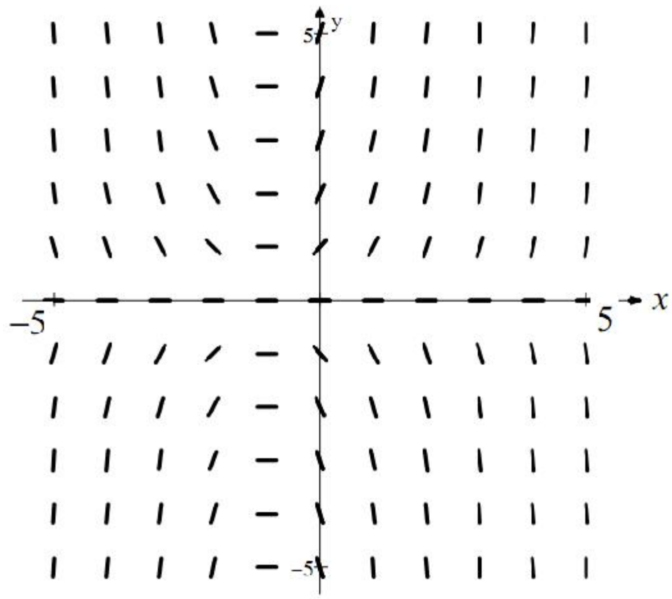
(E) $y = -\sqrt{\frac{2x^3}{3} - 14}$

4.

The function f is twice differentiable with $f(2) = 1$, $f'(2) = 4$, and $f''(2) = 3$. What is the value of the approximation of $f(1.9)$ using the line tangent to the graph of f at $x = 2$?

(A) 0.4 (B) 0.6 (C) 0.7 (D) 1.3 (E) 1.4

5.



Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = xy$

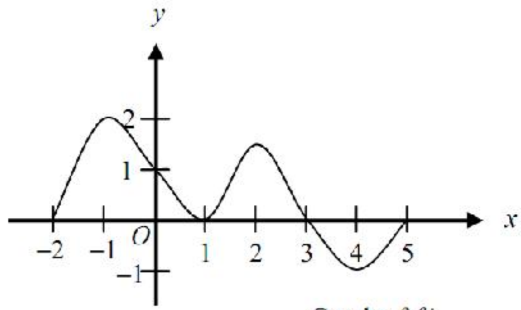
(B) $\frac{dy}{dx} = xy - y$

(C) $\frac{dy}{dx} = xy + y$

(D) $\frac{dy}{dx} = xy + x$

(E) $\frac{dy}{dx} = (x+1)^3$

6.



Graph of f'

The graph of f', the derivative f, is shown above for $-2 \le x \le 5$. On what intervals is f increasing?

(A) $[-2, 1]$ only

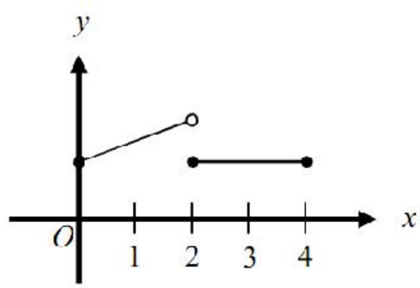
(B) $[-2, 3]$

(C) $[3, 5]$ only

(D) $[0, 1.5]$ and $[3, 5]$

(E) $[-2, -1]$, $[1, 2]$, and $[4, 5]$

7.



Graph of f

The figure above shows the graph of a function f with domain $0 \le x \le 4$. Which of the following statements are true?

I. $\lim_{x \to 2^-} f(x)$ exists.

II. $\lim_{x \to 2^-} f(x)$ exists.

III. $\lim_{x \to 2} f(x)$ exists.

(A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, and III

8.

If $\int_{-5}^{2} f(x)\,dx = -17$ and $\int_{5}^{2} f(x)\,dx = -4$, what is the value of $\int_{-5}^{5} f(x)\,dx$?

(A) -21 (B) -13 (C) 0 (D) 13 (E) 21

9.

If $G(x)$ is an antiderivative for $f(x)$ and $G(2) = -7$, then $G(4) =$

(A) $f'(4)$

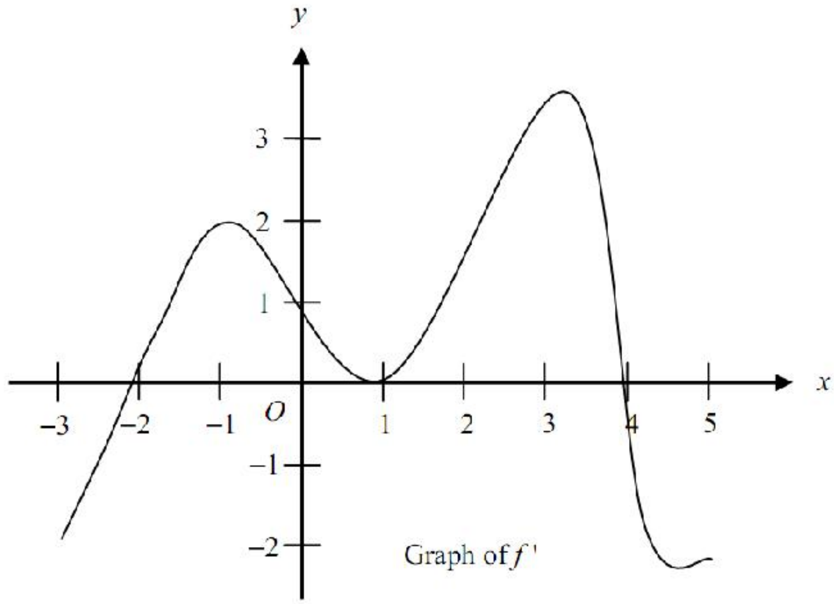
(B) $-7 + f'(4)$

(C) $\int_{2}^{4} f(t)\,dt$

(D) $\int_{2}^{4} \left(-7 + f(t)\right) dt$

(E) $-7 + \int_{2}^{4} f(t)\,dt$

10.



The graph of the derivative of a function f is shown in the figure above. The graph has horizontal tangent lines at $x=-1$, $x=1$, and $x=3$. At which of the following values of x does f have a relative maximum?

(A) -2 only

(B) 1 only

(C) 4 only

(D) -1 and 3 only

(E) -2, 1, and 4

11. (2013, AB-6)

Consider the differential equation $\frac{dy}{dx} = e^y\left(3x^2 - 6x\right)$. Let $y = f(x)$ be the particular solution to the differential equation that passes through $(1, 0)$.

(a) Write an equation for the line tangent to the graph of f at the point $(1, 0)$. Use the tangent line to approximate $f(1.2)$.

(b) Find $y = f(x)$, the particular solution to the differential equation that passes through $(1, 0)$.

(a) pt: $(1,0)$

Slope: $\left.\frac{dy}{dx}\right|_{(1,0)} = 3-6 = -3$

eq: $y(x) = 0 - 3(x-1)$

$f(1.2) \approx y(1.2) = 0 - 3(1.2-1)$

$= -3(.2)$

$= -.6$

(b) $\frac{dy}{dx} = e^y(3x^2 - 6x)$

$\int \frac{1}{e^y}\,dy = \int (3x^2 - 6x)\,dx$ ✓1

$\int e^{-y}\,dy = \int (3x^2 - 6x)\,dx$

$-e^{-y} = x^3 - 3x^2 + C$ ✓2 ✓3 ✓4

$e^{-y} = -x^3 + 3x^2 + C$

$-y = \ln(-x^3 + 3x^2 + C)$

$y = -\ln(-x^3 + 3x^2 + C)$

At $(1,0)$: $0 = -\ln(-1+3+C)$ ✓5

$e^0 = e^{\ln(2+C)}$

$1 = 2 + C$

$C = -1$ ✓6

So, $y = -\ln(-x^3 + 3x - 1)$

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Question 6

Consider the differential equation $\frac{dy}{dx} = e^y\left(3x^2 - 6x\right)$. Let $y = f(x)$ be the particular solution to the differential equation that passes through $(1, 0)$.

(a) Write an equation for the line tangent to the graph of f at the point $(1, 0)$. Use the tangent line to approximate $f(1.2)$.

(b) Find $y = f(x)$, the particular solution to the differential equation that passes through $(1, 0)$.

(a) $\left.\frac{dy}{dx}\right|_{(x,\, y)=(1,\, 0)} = e^0\left(3\cdot 1^2 - 6\cdot 1\right) = -3$

An equation for the tangent line is $y = -3(x-1)$.

$f(1.2) \approx -3(1.2-1) = -0.6$

$3 : \begin{cases} 1 : \frac{dy}{dx} \text{ at the point } (x, y) = (1, 0) \\ 1 : \text{tangent line equation} \\ 1 : \text{approximation} \end{cases}$

(b) $\frac{dy}{e^y} = \left(3x^2 - 6x\right) dx$

$\int \frac{dy}{e^y} = \int \left(3x^2 - 6x\right) dx$

$-e^{-y} = x^3 - 3x^2 + C$

$-e^{-0} = 1^3 - 3\cdot 1^2 + C \Rightarrow C = 1$

$-e^{-y} = x^3 - 3x^2 + 1$

$e^{-y} = -x^3 + 3x^2 - 1$

$-y = \ln\left(-x^3 + 3x^2 - 1\right)$

$y = -\ln\left(-x^3 + 3x^2 - 1\right)$

Note: This solution is valid on an interval containing $x = 1$ for which $-x^3 + 3x^2 - 1 > 0$.

$6 : \begin{cases} 1 : \text{separation of variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: max $3/6$ [1-2-0-0-0] if no constant of integration

Note: $0/6$ if no separation of variables

4. The function f is defined by $f(x) = \sqrt{25 - x^2}$ for $-5 \le x \le 5$.

   (a) Find $f'(x)$.

   (b) Write an equation for the line tangent to the graph of f at $x = -3$.

   (c) Let g be the function defined by $g(x) = \begin{cases} f(x) & \text{for } -5 \le x \le -3 \\ x + 7 & \text{for } -3 < x \le 5. \end{cases}$

   Is g continuous at $x = -3$? Use the definition of continuity to explain your answer.

   (d) Find the value of $\int_0^5 x\sqrt{25 - x^2}\, dx$.

Question 4

The function f is defined by $f(x) = \sqrt{25 - x^2}$ for $-5 \le x \le 5$.

(a) Find $f'(x)$.

(b) Write an equation for the line tangent to the graph of f at $x = -3$.

(c) Let g be the function defined by $g(x) = \begin{cases} f(x) & \text{for } -5 \le x \le -3 \\ x + 7 & \text{for } -3 < x \le 5. \end{cases}$

Is g continuous at $x = -3$? Use the definition of continuity to explain your answer.

(d) Find the value of $\int_0^5 x\sqrt{25 - x^2}\, dx$.

Solution	Points
(a) $f'(x) = \frac{1}{2}\left(25 - x^2\right)^{-1/2}(-2x) = \frac{-x}{\sqrt{25 - x^2}}, \quad -5 < x < 5$	$2 : f'(x)$
(b) $f'(-3) = \frac{3}{\sqrt{25 - 9}} = \frac{3}{4}$ $f(-3) = \sqrt{25 - 9} = 4$ An equation for the tangent line is $y = 4 + \frac{3}{4}(x + 3)$.	$2 : \begin{cases} 1 : f'(-3) \\ 1 : \text{answer} \end{cases}$
(c) $\lim_{x \to -3^-} g(x) = \lim_{x \to -3^-} f(x) = \lim_{x \to -3^-} \sqrt{25 - x^2} = 4$ $\lim_{x \to -3^+} g(x) = \lim_{x \to -3^+} (x + 7) = 4$ Therefore, $\lim_{x \to -3} g(x) = 4$. $g(-3) = f(-3) = 4$ So, $\lim_{x \to -3} g(x) = g(-3)$. Therefore, g is continuous at $x = -3$.	$2 : \begin{cases} 1 : \text{considers one-sided limits} \\ 1 : \text{answer with explanation} \end{cases}$
(d) Let $u = 25 - x^2 \Rightarrow du = -2x\,dx$ $\int_0^5 x\sqrt{25 - x^2}\, dx = -\frac{1}{2}\int_{25}^{0} \sqrt{u}\, du$ $= \left[-\frac{1}{2} \cdot \frac{2}{3} u^{3/2}\right]_{u=25}^{u=0}$ $= -\frac{1}{3}(0 - 125) = \frac{125}{3}$	$3 : \begin{cases} 2 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$