Taylor Series & Polynomials MC Review

Select the correct capital letter. <u>NO CALCULATOR</u> unless specified otherwise.

_____ 1. Let $T_5(x) = 3x^2 - 5x^3 + 7x^4 + 3x^5$ be the fifth-degree Taylor polynomial for the function *f* about $x = 0$. What is the value of $f'''(0)$?

(A) -30 (B) -15 (C) -5 (D) $-\frac{5}{6}$ (E) $-\frac{1}{6}$

_____ 2. For what integer *k*, $k > 1$, will both $\sum_{n=1}^{\infty} \frac{(-1)^{kn}}{n}$ and $\sum_{n=1}^{\infty} \left(\frac{k}{4}\right)^n$ converge?

(A) 6 (B) 5 (C) 4 (D) 3 (E) 2

_____ 3. (<u>Calculator Permitted</u>) The Taylor series for $\ln x$, centered at $x = 1$, is $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(x-1)^n}{n}$. Let *f* be the function given by the sum of the first three nonzero terms of this series. The maximum value of $\left|\ln x - f(x)\right|$ for $0.3 \le x \le 1.7$ is which of the following?

(A) 0.030 (B) 0.039 (C) 0.145 (D) 0.153 (E) 0.529

_____ 4. What are the values of *x* for which the series $\sum_{n=1}^{\infty} \frac{(x+2)^n}{\sqrt{n}}$ converges?

(A) $-3 < x < -1$ (B) $-3 \le x < -1$ (C) $-3 \le x \le -1$ (D) $-1 \le x < 1$ (E) $-1 \le x \le 1$

_____ 5. (Calculator Permitted) The graph of the function represented by the Maclaurin series $1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots + \frac{(-1)^n x^n}{n!} + \cdots$ intersects the graph of $y = x^3$ at $x =$

(A) 0.773 (B) 0.865 (C) 0.929 (D) 1.000 (E) 1.857

_____ 6. Which of the following sequences converge?

I. $\left\{\frac{5n}{2n-1}\right\}$ II. $\left\{\frac{e^n}{n}\right\}$ III. $\left\{\frac{e^n}{1+e^n}\right\}$

(A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, and III

_____ 7. What is the approximation of the value of $\sin 1$ obtained by using the fifth-degree Taylor polynomial about $x=0$ for $\sin x$?

(A) $1-\frac{1}{2}+\frac{1}{24}$ (B) $1-\frac{1}{2}+\frac{1}{4}$ (C) $1-\frac{1}{3}+\frac{1}{5}$ (D) $1-\frac{1}{4}+\frac{1}{8}$ (E) $1-\frac{1}{6}+\frac{1}{120}$

_____ 8. Which of the following series converge?

I. $\sum_{n=1}^{\infty}\frac{n}{n+2}$ II. $\sum_{n=1}^{\infty}\frac{\cos(n\pi)}{n}$ III. $\sum_{n=1}^{\infty}\frac{1}{n}$

(A) None (B) II only (C) III only (D) I and II only (E) I and III only

_____ 9. If $\lim_{b\to\infty}\int_1^b \frac{dx}{x^p}$ is finite, then which of the following must be true?

(A) $\sum_{n=1}^{\infty}\frac{1}{n^p}$ converges (B) $\sum_{n=1}^{\infty}\frac{1}{n^p}$ diverges (C) $\sum_{n=1}^{\infty}\frac{1}{n^{p-2}}$ converges

(D) $\sum_{n=1}^{\infty}\frac{1}{n^{p-1}}$ converges (E) $\sum_{n=1}^{\infty}\frac{1}{n^{p+1}}$ diverges

_____ 10. If $\sum_{n=0}^{\infty} a_n x^n$ is a Taylor series that converges to $f(x)$ for all real *x*, then $f'(1) =$

(A) 0 (B) a_1 (C) $\sum_{n=0}^{\infty} a_n$ (D) $\sum_{n=1}^{\infty} n a_n$ (E) $\sum_{n=1}^{\infty} n a_n^{n-1}$

_____ 11. What is the value of $\sum_{n=1}^{\infty} \frac{2^{n+1}}{3^n}$?

(A) 1 (B) 2 (C) 4 (D) 6 (E) The series diverges

_____ 12. The Maclaurin series for $\frac{1}{1-x}$ is $\sum_{n=0}^{\infty} x^n$. Which of the following is a power series expansion for $\frac{x^2}{1-x^2}$?

(A) $1+x^2+x^4+x^6+x^8+\cdots$ (B) $x^2+x^3+x^4+x^5+\cdots$ (C) $x^2+2x^3+3x^4+4x^5+\cdots$

(D) $x^2+x^4+x^6+x^8+\cdots$ (E) $x^2-x^4+x^6-x^8+\cdots$

_____ 13. A function *f* has a Maclaurin series given by $\frac{x^4}{2!}+\frac{x^5}{3!}+\frac{x^6}{4!}+\cdots+\frac{x^{n+3}}{(n+1)!}+\cdots$. Which of the following is an expression for $f(x)$?

(A) $-3x\sin x+3x^2$ (B) $-\cos\left(x^2\right)+1$ (C) $-x^2\cos x+x^2$

(D) $x^2e^x-x^3-x^2$ (E) $e^{x^2}-x^2-1$

_____ 14. Which of the following series diverge?

I. $\sum_{n=0}^{\infty}\left(\frac{\sin 2}{\pi}\right)^n$ II. $\sum_{n=1}^{\infty}\frac{1}{\sqrt[3]{n}}$ III. $\sum_{n=1}^{\infty}\left(\frac{e^n}{e^n+1}\right)$

(A) III only (B) I and II only (C) I and III only (D) II and III only (E) I, II, and III

_____ 15. What is the coefficient of x^2 in the Taylor series for $\frac{1}{(1+x)^2}$ about $x=0$?

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) 1 (D) 3 (E) 6

_____ 16. The sum of the infinite geometric series $\frac{3}{2}+\frac{9}{16}+\frac{27}{128}+\frac{81}{1024}+\cdots$ is

(A) 1.60 (B) 2.35 (C) 2.40 (D) 2.45 (E) 2.50

_____ 17. What are all values of *x* for which the series $\sum_{n=1}^{\infty}\frac{(x-2)^n}{n3^n}$ converges?

(A) $-3\le x\le 3$ (B) $-3<x<3$ (C) $-1<x\le 3$ (D) $-1\le x\le 5$ (E) $-1\le x<5$

_____ 18. The Taylor series for $\sin x$ about $x=0$ is $x-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots$. If *f* is a function such that $f'(x)=\sin\left(x^2\right)$, then the coefficient of x^7 in the Taylor series for $f(x)$ about $x=0$ os

(A) $\frac{1}{7!}$ (B) $\frac{1}{7}$ (C) 0 (D) $-\frac{1}{42}$ (E) $-\frac{1}{7!}$

_____ 19. $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n x^n}{n!}$ is the Taylor series about zero for which of the following functions?

(A) $\sin x$ (B) $\cos x$ (C) e^x (D) e^{-x} (E) $\ln(1+x)$

_____ 20. For what values of x does the series $1+2^x+3^x+4^x+\cdots+n^x+\cdots$ converge?

(A) No values of x (B) $x<-1$ (C) $x\geq -1$ (D) $x>-1$ (E) All values of x

_____ 21. The complete interval of convergence of the series $\displaystyle\sum_{k=1}^{\infty}\frac{(x+1)^k}{k^2}$ is

(A) $0<x<2$ (B) $0\leq x\leq 2$ (C) $-2<x\leq 0$ (D) $-2\leq x<0$ (E) $-2\leq x\leq 0$

_____ 21. For $-1<x<1$, if $f\left(x\right)=\sum_{n=1}^{\infty}\frac{\left(-1\right)^{n+1}x^{2n-1}}{2n-1}$, then $f'\left(x\right)=$

(A) $\sum_{n=1}^{\infty}\left(-1\right)^{n+1}x^{2n-2}$ (B) $\sum_{n=1}^{\infty}\left(-1\right)^{n}x^{2n-2}$ (C) $\sum_{n=1}^{\infty}\left(-1\right)^{2n}x^{2n}$ (D) $\sum_{n=1}^{\infty}\left(-1\right)^{n}x^{2n}$ (E) $\sum_{n=1}^{\infty}\left(-1\right)^{n+1}x^{2n}$

_____ 22. The coefficient of x^3in the Taylor series fo e^{3x} about $x=0$ is

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{3}{2}$ (E) $\frac{9}{2}$

_____ 23. $\sum_{i=n}^{\infty}\left(\frac{1}{3}\right)^{i}=$

(A) $\frac{3}{2}-\left(\frac{1}{3}\right)^{n}$ (B) $\frac{3}{2}\left[1-\left(\frac{1}{3}\right)^{n}\right]$ (B) $\frac{3}{2}\left(\frac{1}{3}\right)^{n}$ (D) $\frac{2}{3}\left(\frac{1}{3}\right)^{n}$ (E) $\frac{2}{3}\left(\frac{1}{3}\right)^{n+1}$

_____ 24. Which of the followign series converge?

$$\text{I. } \sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{2n+1} \qquad \text{II. } \sum_{n=1}^{\infty}\frac{1}{n}\left(\frac{3}{2}\right)^{n} \qquad \text{III. } \sum_{n=2}^{\infty}\frac{1}{n\ln n}$$

(A) I only (B) II only (C) III only (D) I and III only (E) I, II, and III

_____ 25. If $s_n = \left(\frac{(5+n)^{100}}{5^{n+1}}\right)\left(\frac{5^n}{(4+n)^{100}}\right)$, to what number does the sequence $\{s_n\}$ converge?

(A) $\frac{1}{5}$ (B) 1 (C) $\frac{5}{4}$ (D) $\left(\frac{5}{4}\right)^{100}$ (E) The sequence does not converge

_____ 26. (Calculator Permitted) If $f(x) = \sum_{k=1}^{\infty}\left(\sin^2 x\right)^k$, then $f(1)$ is

(A) 0.369 (B) 0.585 (C) 2.400 (D) 2.426 (E) 3.426

_____ 27. Let the function given by $f(x)=\ln(3-x)$. The third-degree Taylor polynomial for f about $x=2$ is

(A) $-(x-2)+\frac{(x-2)^2}{2}-\frac{(x-2)^3}{3}$ (B) $-(x-2)-\frac{(x-2)^2}{2}-\frac{(x-2)^3}{3}$ (C) $(x-2)+(x-2)^2+(x-2)^3$

(D) $(x-2)+\frac{(x-2)^2}{2}+\frac{(x-2)^3}{3}$ (E) $(x-2)-\frac{(x-2)^2}{2}+\frac{(x-2)^3}{3}$

_____ 28. (Calculator Permitted) Suppose a function f is approximated with a fourth-degree Taylor polynomial about $x=1$. If the maximum value of the fifth derivative between $x=1$ and $x=3$ is 0.01, that is, $\left|f^{(5)}(x)\right|<0.01$, then the maximum error incurred using this approximation to compute $f(3)$ is

(A) 0.054 (B) 0.0054 (C) 0.26667 (D) 0.02667 (E) 0.00267