1. B
2. A
3. A
4. D
5. D
6. D
7. C
8. D
9. B
10. C

01

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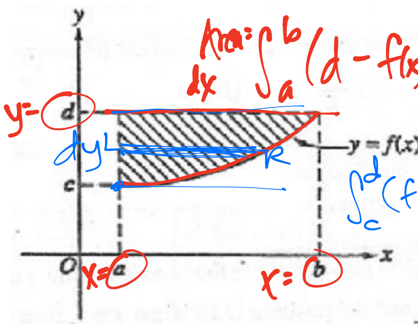
BC Review 01, **No** Calculator Permitted



Area: $\int_a^b (d - f(x))\,dx$ dx

$y = d$ dy   R $x = a$ $x = b$

$x = f(y)$ $\int_c^d (f(y) - a)\,dy$

$\int_a^b f'(x)\,dx = f(x)\Big|_a^b$

$f(b) - f(a)$

$\int_a^b f(x)\,dx = F(x)\Big|_a^b$

$F(b) - F(a)$

1. Which of the following represents the area of the shaded region in the figure above?

(A) $\int_c^d f(y)\,dy$ (B) $\int_a^b (d - f(x))\,dx$ (C) $f'(b) - f'(a)$

(D) $(b-a)[f(b) - f(a)]$ (E) $(d-c)[f(b) - f(a)]$

$\frac{dy}{dx} = \frac{2y - 3x}{3x - 5y}\left(\frac{-1}{-1}\right)$

$\frac{3x - 2y}{5y - 3x}$

$\frac{dy}{dx}(3x + 6y^2) = -3x^2 - 3y$

$\frac{dy}{dx} = \frac{-3(x^2 + y)}{3(x + 2y^2)}$

$\frac{d}{dx}: 3x^2 + 3y + 3x \cdot \frac{dy}{dx} + 6y^2\frac{dy}{dx} = 0$

2. If $x^3 + 3xy + 2y^3 = 17$, then in terms of x and y, $\frac{dy}{dx} =$

(A) $-\frac{x^2 + y}{x + 2y^2}$ (B) $-\frac{x^2 + y}{x + y^2}$ (C) $-\frac{x^2 + y}{x + 2y}$ (D) $-\frac{x^2 + y}{2y^2}$ (E) $-\frac{x^2}{1 + 2y^2}$

$(3)(\frac{1}{3})$

3. $\int \frac{3x^2}{\sqrt{x^3 + 1}}\,dx =$ $\int (3x^2)(x^3 + 1)^{-1/2}\,dx = 2(x^3 + 1)^{1/2} + C$

(A) $2\sqrt{x^3 + 1} + C$ (B) $\frac{3}{2}\sqrt{x^3 + 1} + C$ (C) $\sqrt{x^3 + 1} + C$ (D) $\ln\sqrt{x^3 + 1} + C$ (E) $\ln(x^3 + 1) + C$

what is the rel min? (A) 3

4. For what value of ~~x~~ does the function $f(x)=(x-2)(x-3)^2$ have a relative ~~maximum~~?

critical values ers

(A) -3 (B) $-\frac{7}{3}$ (C) $-\frac{5}{2}$ (D) $\frac{7}{3}$ (E) $\frac{5}{2}$

$f'=0$ or $f'=DNE$

$f' = 1\cdot(x-3)^2+(x-2)\cdot 2(x-3)\cdot 1=0$

$(x-3)\left[x-3+2x-4\right]=0$

$(x-3)(3x-7)=0$

$x=3,\ x=\frac{7}{3}$

x	0	7/3	2.9	3	4
f'	+		−		+

5. If $f(x)=\sin\left(\frac{x}{2}\right)$, then there exists a number c in the interval $\frac{\pi}{2}<x<\frac{3\pi}{2}$ that satisfies the conclusion of the Mean Value Theorem. Which of the following could be c?

MVT

(A) $\frac{2\pi}{3}$ (B) $\frac{3\pi}{4}$ (C) $\frac{5\pi}{6}$ (D) π (E) $\frac{3\pi}{2}$

$f'=\frac{f\left(\frac{3\pi}{2}\right)-f\left(\frac{\pi}{2}\right)}{\frac{3\pi}{2}-\frac{\pi}{2}}$

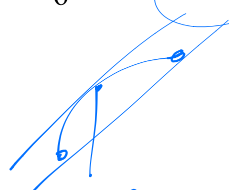
$\frac{1}{2}\cos\left(\frac{x}{2}\right)=\frac{\sqrt{2}/2-\sqrt{2}/2}{\pi}$

$\frac{1}{2}\cos\left(\frac{x}{2}\right)=0$

$\cos\left(\frac{x}{2}\right)=0$

$\frac{x}{2}=\frac{\pi}{2}\rightarrow x=\pi$

$\frac{x}{2}=\frac{3\pi}{2}\rightarrow x=3\pi$



x=c

$[0,2]$

$x=-1,0,1$

(A) $x=0,1$ (B) $x=1$

6. If $f(x)=(x-1)^2\sin x$, then $f'(0)=$

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

$f'=2(x-1)\cdot 1\cdot \cancel{\sin x}+(x-1)^2\cdot\cos x$

$f'(0)=1$

7. The acceleration of a particle moving along the x-axis at time t is given by $a(t)=6t-2$. If the velocity is 25 when $t=3$ and the position is 10 when $t=1$, then the position $x(t)=$

(A) $9t^2+1$ (B) $3t^2-2t+4$ (C) t^3-t^2+4t+6 (D) $t^3-t^2+9t-20$ (E) $36t^3-4t^2-77t+55$

$V(t)=3t^2-2t+C$

$25=27-6+C$

$C=4$

$V(t)=3t^2-2t+4$

$x(t)=t^3-t^2+4t+C$

2nd FTOC $\cos(2\pi x)\cdot 1 - \cos(2\pi\cdot 0)\cdot 0$

8. $\dfrac{d}{dx}\displaystyle\int_0^x \cos(2\pi u)\,du$ is

$\cos(2\pi x)$

(A) 0 (B) $\dfrac{1}{2\pi}\sin x$ (C) $\dfrac{1}{2\pi}\cos(2\pi x)$ (D) $\cos(2\pi x)$ (E) $2\pi\cos(2\pi x)$

Brook Taylor

9. $\displaystyle\int xf(x)\,dx =$

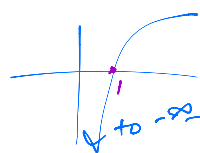
dv u

(A) $xf(x)-\displaystyle\int xf'(x)dx$ (B) $\dfrac{x^2}{2}f(x)-\displaystyle\int\dfrac{x^2}{2}f'(x)dx$ (C) $xf(x)-\dfrac{x^2}{2}f(x)+C$

(D) $xf(x)-\displaystyle\int f'(x)dx$ (E) $\displaystyle\int\dfrac{x^2}{2}f(x)dx$

u	dv	+/-
$f(x)$	x	+
$f'(x)$	$\frac{1}{2}x^2$	−
$f''(x)$		

$\frac{1}{2}x^2 f(x) - \int \frac{1}{2}x^2 \cdot f'(x)\,dx$



D: $\{x \mid x>0\}$

$(0,\infty)$

10. What is the minimum value of $f(x)=x\ln x$?

(A) $-e$ (B) -1 (C) $-\dfrac{1}{e}$ (D) 0 (E) $f(x)$ has no minimum value.

$f'(x) = 1\cdot\ln x + x\cdot\left(\frac{1}{x}\right) = 0$

$\boxed{\ln x + 1} = 0$

$e^{\ln x} = e^{-1}$

a

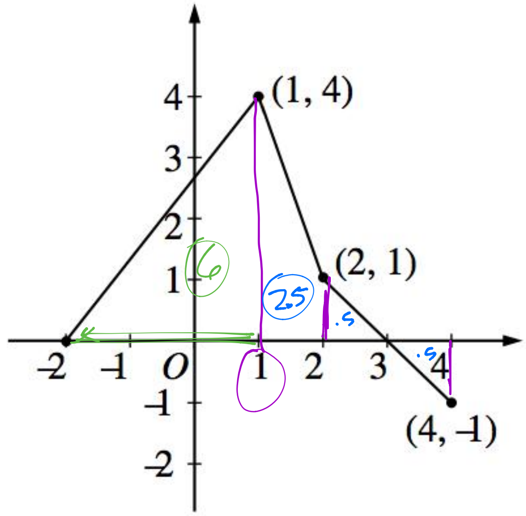
$x = e^{-1} = \frac{1}{e}$

$f\left(\frac{1}{e}\right) = \frac{1}{e}\cdot\ln\left(\frac{1}{e}\right) = e^{-1}\cdot\ln\left(e^{-1}\right)$

$= -e^{-1} = -\frac{1}{e}$

$\frac{1}{e}$

x	.036…	1
f'	−	+



11. (1999, AB-5) The graph of the function f, consisting of three line segments, is shown above. Let $g(x) = \int_1^x f(t)\,dt$.

(a) Compute $g(4)$ and $g(-2)$.

$g(4) = \int_1^4 f(t)\,dt = 2.5 + .5 - .5 = 2.5$

$g(-2) = \int_1^{-2} f(t)dt = -6$

(b) Find the instantaneous rate of change of g, with respect to x, at $x = 1$.

$\rightarrow g'(x) = f(x)$

$\rightarrow g'(1) = f(1) = 4$

(c) Find the absolute minimum value of g on the closed interval $[-2, 4]$. Justify your answer.

EVT

$g'(x) = f(x) = 0$, $x = 3$

$g'(x) = f(x) = \text{DNE}$

$g(-2) = -6$

$g(4) = 2.5$

$g(3) = \int_1^3 f(t)\,dt = 2.5 + .5 = 3$

by the EVT,

so the abs min of $g(x)$ is -6.

(d) The second derivative of g is not defined at $x = 1$ and $x = 2$. How many of these values are x-coordinates of points of inflection of the graph of g? Justify your answer.

① $g'' = 0$ or $g'' = \text{DNE} \rightarrow$ p.i.v.

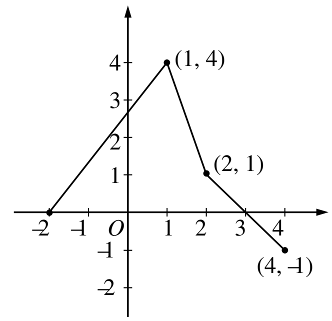
② sign change at $x =$ p.i.v.

$g'(x) = f(x)$

$g''(x) = f'(x) = \text{slopes of } f(x)$

$g''(x) = \text{DNE}$ AT $x = 1, 2$

5. The graph of the function f, consisting of three line segments, is given above. Let $g(x) = \int_1^x f(t)\,dt$.



(a) Compute $g(4)$ and $g(-2)$.

(b) Find the instantaneous rate of change of g, with respect to x, at $x = 1$.

(c) Find the absolute minimum value of g on the closed interval $[-2, 4]$. Justify your answer.

(d) The second derivative of g is not defined at $x = 1$ and $x = 2$. How many of these values are x–coordinates of points of inflection of the graph of g? Justify your answer.

(a) $g(4) = \int_1^4 f(t)\,dt = \frac{3}{2} + 1 + \frac{1}{2} - \frac{1}{2} = \frac{5}{2}$ $g(-2) = \int_1^{-2} f(t)\,dt = -\frac{1}{2}(12) = -6$	**2** { 1: $g(4)$ 1: $g(-2)$
(b) $g'(1) = f(1) = 4$	**1**: answer
(c) g is increasing on $[-2, 3]$ and decreasing on $[3, 4]$. Therefore, g has absolute minimum at an endpoint of $[-2, 4]$. Since $g(-2) = -6$ and $g(4) = \frac{5}{2}$, the absolute minimum value is -6.	**3** { 1: interior analysis 1: endpoint analysis 1: answer
(d) One; $x = 1$ On $(-2, 1)$, $g''(x) = f'(x) > 0$ On $(1, 2)$, $g''(x) = f'(x) < 0$ On $(2, 4)$, $g''(x) = f'(x) < 0$ Therefore $(1, g(1))$ is a point of inflection and $(2, g(2))$ is not.	**3** { 1: choice of $x = 1$ only 1: show $(1, g(1))$ is a point of inflection 1: show $(2, g(2))$ is not a point of inflection

12. (1998, AB-4) Let f be a function with $f(1)=4$ such that for all points (x, y) on the graph of f the slope is given by $\frac{3x^2+1}{2y} = \frac{dy}{dx}$

(a) Find the slope of the graph of f at the point where $x=1$.

$$\left.\frac{dy}{dx}\right|_{(1,4)} = \frac{3+1}{2(4)} = \frac{1}{2}$$

(b) Write an equation for the line tangent to the graph of f at $x=1$, and use it to approximate $f(1.2)$.

$f(1)=4$

$f'(1,4)=\frac{1}{2}$

eq: $y = 4+\frac{1}{2}(x-1) = T_1(x)$

$f(1.2) \approx T_1(1.2) = 4+\frac{1}{2}(.2) = 4.1$

(c) Find $f(x)$ by solving the separable differential equation $\frac{dy}{dx} = \frac{3x^2+1}{2y}$ with the initial condition $f(1)=4$.

$2y\,dy = (3x^2+1)dx$

(d) Use your solution from part (c) to find $f(1.2)$.

1998 AP Calculus AB Scoring Guidelines

4. Let f be a function with $f(1) = 4$ such that for all points (x, y) on the graph of f the slope is given by $\dfrac{3x^2+1}{2y}$.
   (a) Find the slope of the graph of f at the point where $x = 1$.
   (b) Write an equation for the line tangent to the graph of f at $x = 1$ and use it to approximate $f(1.2)$.
   (c) Find $f(x)$ by solving the separable differential equation $\dfrac{dy}{dx} = \dfrac{3x^2+1}{2y}$ with the initial condition $f(1) = 4$.
   (d) Use your solution from part (c) to find $f(1.2)$.

(a) $\dfrac{dy}{dx} = \dfrac{3x^2+1}{2y}$

$$\left.\frac{dy}{dx}\right|_{\substack{x=1\\y=4}} = \frac{3+1}{2\cdot 4} = \frac{4}{8} = \frac{1}{2}$$

1: answer

(b) $y - 4 = \dfrac{1}{2}(x-1)$

$f(1.2) - 4 \approx \dfrac{1}{2}(1.2-1)$

$f(1.2) \approx 0.1 + 4 = 4.1$

2 $\begin{cases} 1: & \text{equation of tangent line} \\ 1: & \text{uses equation to approximate } f(1.2) \end{cases}$

(c) $2y\,dy = (3x^2+1)\,dx$

$\displaystyle\int 2y\,dy = \int (3x^2+1)\,dx$

$y^2 = x^3 + x + C$

$4^2 = 1 + 1 + C$

$14 = C$

$y^2 = x^3 + x + 14$

$y = \sqrt{x^3+x+14}$ is branch with point $(1, 4)$

$f(x) = \sqrt{x^3+x+14}$

5 $\begin{cases} 1: & \text{separates variables} \\ 1: & \text{antiderivative of } dy \text{ term} \\ 1: & \text{antiderivative of } dx \text{ term} \\ 1: & \text{uses } y = 4 \text{ when } x = 1 \text{ to pick one function out of a family of functions} \\ 1: & \text{solves for } y \\ & 0/1 \text{ if solving a linear equation in } y \\ & 0/1 \text{ if no constant of integration} \end{cases}$

Note: max 0/5 if no separation of variables

Note: max 1/5 [1-0-0-0-0] if substitutes value(s) for x, y, or dy/dx before antidifferentiation

(d) $f(1.2) = \sqrt{1.2^3 + 1.2 + 14} \approx 4.114$

1: answer, from student's solution to the given differential equation in (c)

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