1. C
2. B
3. C
4. D
5. C
6. C
7. B
8. E
9. E
10. A

5

BC Review 05, Use your calculator ONLY on #2.
Do all work on separate notebook paper

1. If $\frac{dy}{dx} = y\sec^2 x$ and $y = 5$ when $x = 0$, then $y =$

(A) $e^{\tan x} + 4$ (B) $e^{\tan x} + 5$ (C) $5e^{\tan x}$ (D) $\tan x + 5$ (E) $\tan x + 5e^x$

2. (Calculator Permitted) The average value of the function $f(x) = e^{-x^2}$ on the closed interval $[-1,1]$ is

(A) 0.70 (B) 0.75 (C) 0.80 (D) 0.85 (E) 0.90

3. If $\frac{dy}{dx} = \frac{1}{x}$, then the average rate of change of y with respect to x on the closed interval $[1,4]$ is

(A) $-\frac{1}{4}$ (B) $\frac{1}{2}\ln 2$ (C) $\frac{2}{3}\ln 2$ (D) $\frac{2}{5}$ (E) 2

4. Given the differential equation $\frac{dy}{dx} = \frac{1}{x+1}$ and $y(0) = 0$. An approximation of $y(1)$ using Euler's method with two steps and step size $\Delta x = 0.5$ is

(A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) $\frac{5}{6}$ (E) $\frac{9}{10}$

5. If a population of wolves grows according to the differential equation $\frac{dN}{dt} = 0.05N - 0.0005N^2$, where N is the number of wolves and t is measured in years, then $\lim_{t\to\infty} N(t) =$

(A) 50 (B) 75 (C) 100 (D) 150 (E) 200

6. $\lim_{h\to 0} \frac{3\left(\frac{1}{2}+h\right)^5 - 3\left(\frac{1}{2}\right)^5}{h} =$

$f = 3x^5$
$f' = 15x^4 \Rightarrow 15\left(\frac{1}{2}\right)^4 \quad \frac{15}{2^4}$

(A) 0 (B) 1 (C) $\frac{15}{16}$ (D) the limit does not exist (E) the limit cannot be determined

7. If f is continuous for $a \le x \le b$ and differentiable for $a < x < b$, which of the following could be false?

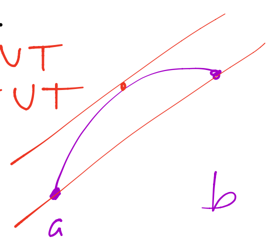
MVT (A) $\boxed{f'(c)} = \boxed{\frac{f(b)-f(a)}{b-a}}$ for some c such that $a < c < b$.

→ (B) $f'(c) = 0$ for some c such that $a < c < b$.

(C) f has a minimum value on $a \le x \le b$. EVT

(D) f has a maximum value on $a \le x \le b$. EVT

(E) $\int_a^b f(x)\,dx$ exists



C → I

D

8. $\int_1^e \left(\frac{x^2-1}{x}\right)dx =$

$\int_1^e \left(\frac{x^2}{x} - \frac{1}{x}\right)$

(A) $e - \frac{1}{e}$ (B) $e^2 - e$ (C) $\frac{e^2}{2} - e + \frac{1}{2}$ (D) $e^2 - 2$ (E) $\frac{e^2}{2} - \frac{3}{2}$

$\int_1^e \left(x - \frac{1}{x}\right)dx = \frac{1}{2}x^2 - \ln x \Big|_1^e$

$\left(\frac{e^2}{2} - 1\right) - \left(\frac{1}{2}\right)$

$\frac{e^2}{2} - \frac{3}{2}$

9. $\int_0^\pi x\cos x\,dx =$

(A) $\frac{\pi}{3} - \frac{4}{3}$ (B) π (C) $\frac{\pi}{2} - 1$ (D) $\frac{\pi}{4} - \frac{3}{2}$ (E) -2

pt. $(5, 7)$

$y = 7 + \frac{5}{7}(x - 5)$

$y(0) = 7 - \frac{25}{7}$

10. Determine the y-intercept of the tangent line to the curve $y = \sqrt{x^2 + 24}$ at $x = 5$.

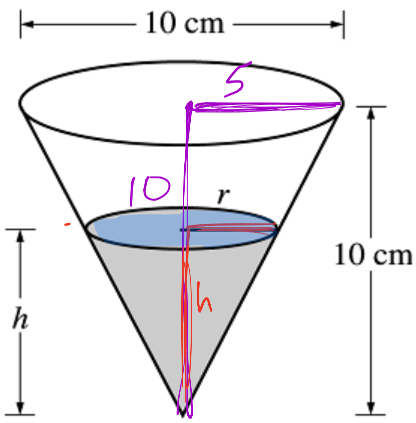
(A) $\frac{24}{7}$ (B) $-\frac{72}{49}$ (C) $\frac{48}{49}$ (D) $\frac{44}{7}$ (E) $\frac{88}{49}$

$y' = \frac{1}{2}(x^2+24)^{-1/2}(2x) = \frac{x}{\sqrt{x^2+24}} \to \frac{5}{7}$

11. (2002-AB5) A container has the shape of an open right circular cone, as shown in the figure on the right. The height of the container is 10 cm, and the diameter of the opening is 10 cm. Water in the container is evaporating so that its depth h is changing at the constant rate of $-\frac{3}{10}$ cm/hr.

$\frac{dh}{dt} = -\frac{3}{10}$

(a) Find the volume V of water in the container when $h = 5$ cm. Indicate units of measure.
(b) Find the rate of change of the volume of water in the container, with respect to time, when $h = 5$ cm. Indicate units of measure.
(c) Show that the rate of change of the volume of water in the container due to evaporation is directly proportional to the exposed surface area of the water. What is the constant of proportionality?



$\frac{10}{5} = \frac{h}{r}$

$2 = \frac{h}{r}$,

$h = 2r$

$r = \frac{1}{2}h$

$\frac{dh}{dt} = 2\frac{dr}{dt}$

$\frac{dr}{dt} = \frac{1}{2}\frac{dh}{dt}$

$V = \frac{\pi}{3}r^2h$

$V(r) = \frac{\pi}{3}r^2(2r)$

$V = \frac{2\pi}{3}r^3$

$V(h) = \frac{\pi}{3}\left(\frac{1}{2}h\right)^2 h$

$V = \frac{\pi}{12}h^3$

(b) $\frac{dV}{dt} = ?$ $h = 5$

$V = \frac{\pi}{12}h^3$

$\frac{d}{dt}: \frac{dV}{dt} = \frac{\pi}{4}h^2 \cdot \frac{dh}{dt}$

When $h=5$: $\frac{dV}{dt} = \frac{\pi}{4}(5^2)\left(-\frac{3}{10}\right)$ cm³/hr

(a) V $h = 5$

$V(5) = \frac{\pi}{12}(5^3) = \frac{125\pi}{12}$ cm³

(c) $\frac{dV}{dt} = K \cdot \pi r^2$

$V = \frac{2\pi}{3}r^3$

$\frac{d}{dt}: \frac{dV}{dt} = 2\pi r^2 \cdot \frac{dr}{dt}$

$\frac{dV}{dt} = \left(2\frac{dr}{dt}\right)\pi r^2$

$K = 2\frac{dr}{dt} = \frac{dh}{dt} = -\frac{3}{10}$

x	-1.5	-1.0	-0.5	0	0.5	1.0	1.5
$f(x)$	-1	-4	-6	-7	-6	-4	-1
$f'(x)$	-7	-5	-3	0	3	5	7

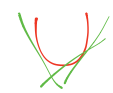
12. (2002-AB6) Let f be a function that is differentiable for all real numbers. The table above gives the values of f and its derivative f' for selected points x in the closed interval $-1.5 \le x \le 1.5$. The second derivative of f has the property that $f''(x) > 0$ for $-1.5 \le x \le 1.5$.

f is ccup   f' is inc

(a) Evaluate $\int_0^{1.5} \left(3f'(x)+4\right)dx$. Show the work that leads to your answer.

$3\cdot f(x) + 4x \Big|_0^{1.5} = (3f(1.5)+6) - (3\cdot f(0)) = 3(-1)+6-3(-7) = 24$

(b) Write an equation of the line tangent to the graph of f at the point where $x = 1$. Use this line to approximate the value of $f(1.2)$. Is this approximation greater than or less than the actual value of $f(1.2)$? Give a reason for your answer.



$f(1) = -4$
$f'(1) = 5$

$T(x) = -4 + 5(x-1)$
$f(1.2) \approx T(1.2) = -4 + 5(1.2-1) = -3$

$f'' > 0$ for all $x \in [-1.5, 1.5]$

(c) Find a positive real number r having the property that there must exist a value c with $0 < c < 0.5$ and $f''(x) = r$. Give a reason for your answer.

(d) Let g be the function given by $g(x) = \begin{cases} 2x^2 - x - 7 & \text{for } x < 0 \\ 2x^2 + x - 7 & \text{for } x \ge 0 \end{cases}$. The graph of g passes through each of the points $(x, f(x))$ given in the table above. Is it possible that f and g are the same function? Give a reason for your answer.

$g'(x) = \begin{cases} 4x - 1, & x < 0 \\ 4x + 1, & x \ge 0 \end{cases}$

$\lim_{x \to 0^-} g'(x) = -1$

$\lim_{x \to 0^+} g'(x) = 1$

Since $-1 \ne 1$, $g(x)$ is not diff'ble at $x = 0$ while $f(x)$ is, ergo, they are not the same function